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Determination of the mean relative photoelastic dispersion factor by means of photoelasticity

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ABSTRACT

This work proposed to determine the mean dispersion factor, $< \boldsymbol{\alpha}_C >$, which is related to the relative photoelastic dispersion coefficient of a photoelastic material, subjected to consecutive external stresses, by means of a direct computational method associated with error theory. An important feature of photoelasticity, related to the light wavelength, is the photoelastic dispersion coefficient, which is determined in traditional methods by indirect statistical processes, as it depends on refractive indices, which are difficult to determine in photoelastic materials. Therefore, finding an alternative form to determine this coefficient is necessary and can help in the processes that involve the use of photoelasticity. The mean dispersion factor obtained was $< \boldsymbol{\alpha}_C > = (8{,}90 \pm 0{,}42) \times 10^{-5}$, which works as a system calibrator to allow the determination of the photoelastic dispersion coefficient of photoelastic materials by photoelasticity, with greater accuracy.

Keywords: photoelasticity; photoelastic materials; birefringence; error theory.

INTRODUCTION

Several photoelasticity techniques and methods can be used to determine the relative photoelastic dispersion coefficient. In this sense, some published works show the relevance of applications in the areas of Physics, Mechanical Engineering, Materials Engineering and Dentistry (DA SILVA, 2017; DA SILVA et. al, 2017; DOBRANSZKI et. al, 2010; RAMESH, 2000; SOARES, 1997).

Photoelasticity is a branch of optics that studies the distribution of stresses and strains in photoelastic materials with the aid of polarized light (FERREIRA, 2003; HECHT, 2002; GUENTER, 1990; PRADO et. al, 2020). Photoelastic materials have the property of temporary birefringence and, mainly, due to their transparency and elasticity, they are widely used in industry for indirect determination of the properties of materials such as iron, steel, concrete, etc. (BREWSTER, 1815; PARTHASARATHI, 2018; TORO et. al, 2017).

An important feature of photoelasticity, related to the wavelength of light, is the relative coefficient of photoelastic dispersion, which is determined, in traditional methods, by indirect statistical processes, as it depends on refractive indices, which are difficult to determine in photoelastic materials. It is essential to search for new methods for the treatment of data obtained by the various optical techniques that already exist and are being improved. In this direction, a computational system capable of processing a large amount of data quickly and efficiently will contribute to the advancement of studies on photoelastic materials (FERREIRA, 2003; HECHT, 2002; GUENTHER, 1990).

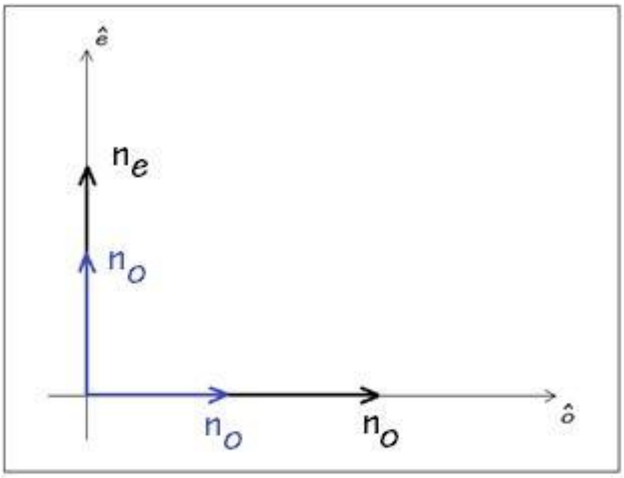
The method proposed in this paper determined the mean dispersion factor, $<\boldsymbol{\alpha_c}>$, related to the relative photoelastic dispersion coefficient of photoelastic samples, through computer programs, from algorithms that performed the partial processes, which automated the method of calculating the mean strains of the evolution of photoelastic fringes, observed by a digital camera, during increasing external stresses produced in the samples, through the linear transmission polariscope technique.

The programs carry out the partial processes from recording the captured video and its separation into frames, through the calculation of mean strains and relating them to the mean strains to the construction of strain versus strain graphs to obtain the mean relative photoelastic dispersion factor.

THEORY

For a photoelastic material that exhibits the effect of temporary birefringence under the action of external stress, the optical stress law relates changes in the refractive index with its plane stress state (TORO et. al, 2017; KUSKE; ROBERTSON, 1974; FERREIRA JUNIOR, 2003). These changes are linearly proportional to the external mechanical stresses on the material and, consequently, these refractive indices are associated with the stresses and strains inside. **Figure 1** shows a scheme of these refractive indices in an infinitesimal element of the material.

Figure 1 – Schematic of refractive indices in the ordinary, and extraordinary, directions and in an infinitesimal element of a photoelastic material.



Source: Da Silva (2017).

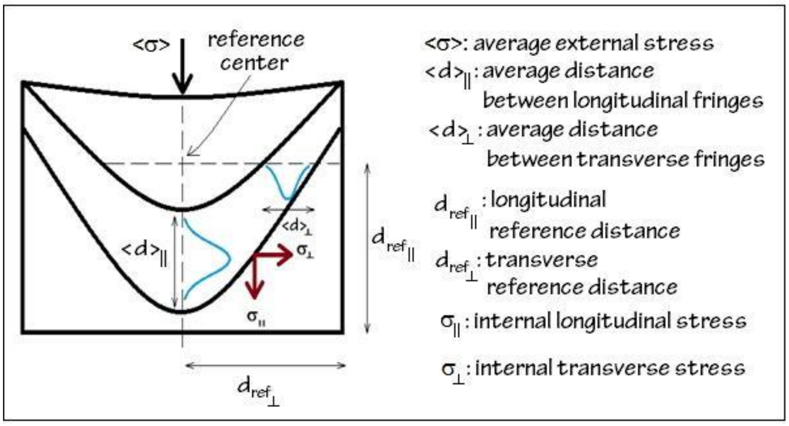
Without external stresses the refractive indices, $\boldsymbol{n_o}$, are equal in both directions. When there is an external stress, the voltage difference promotes a difference in refractive indices in both directions, characterizing the temporary birefringence of the material. In practice, the birefringence of photoelastic materials observed in a polariscope with polychromatic light, which is the polarizer-based instrument for observing this effect, the result is a set of colored and dark fringes in the plane parallel to the direction of the applied external stress. The equation that represents this idea is:

$$\mathrm{n_o} - \mathrm{n_e} = \mathrm{C}\,(\sigma_\mathrm{o} - \sigma_\mathrm{e}), \tag{1}$$

$\boldsymbol{n_e}$ and $\boldsymbol{n_o}$ are the refractive indices in the extraordinary and ordinary directions, respectively, according to the reference system in **Figure 1**. $\boldsymbol{\sigma_e}$ and $\boldsymbol{\sigma_o}$ are the internal stresses in the extraordinary and ordinary directions, respectively. $\mathrm{C} \equiv \mathbf{c_e} - \mathbf{c_o}$ is called the relative optical stress coefficient. In specific cases, this coefficient is considered a wavelength-independent material constant, $\boldsymbol{\lambda}$, but in general cases C is wavelength dependent and $\mathrm{C} = \mathrm{C}(\boldsymbol{\lambda})$ is called birefringence dispersion or photoelastic dispersion.

Figure 2 shows the fringes evolution scheme according to the increase of applied compression, in order to illustrate the distances between the fringes in the transverse and longitudinal directions visible along the sample.

Figure 2 – Fringe evolution scheme for compressive stresses in a photoelastic sample



Source: Own authorship.

Assuming that $\boldsymbol{\sigma}_{\|} - \boldsymbol{\sigma}_{\perp}$ is directly proportional to the mean external stress, $< \sigma >$, and that $\mathrm{n}_{\|} - \mathrm{n}_{\perp}$ is proportional to the difference between the mean relative distances between fringes $\langle \boldsymbol{\varepsilon}_{\|} \rangle - \langle \boldsymbol{\varepsilon}_{\perp} \rangle$, less than a constant $< \boldsymbol{\alpha}_{\mathbf{C}} >$, mean scatter factor, for a given stress, then:

$$<\sigma>_{\mathrm{i}} = \frac{\langle \alpha_C \rangle}{C} \left(\langle \varepsilon_{\|} \rangle_{\mathrm{i}} - \langle \varepsilon_{\perp} \rangle_{\mathrm{i}} \right). \quad (2)$$

The index $\boldsymbol{i}$ represents each stress applied to the sample. Defining $\Delta \langle \boldsymbol{\varepsilon} \rangle_{\mathbf{i}} \equiv \langle \boldsymbol{\varepsilon}_{\|} \rangle_{\mathbf{i}} - \langle \boldsymbol{\varepsilon}_{\perp} \rangle_{\mathbf{i}}$, we have:

$$<\sigma>_{\mathrm{i}} = \frac{\langle \alpha_C \rangle}{C} (\Delta \langle \varepsilon \rangle_{\mathrm{i}}). \quad (3)$$

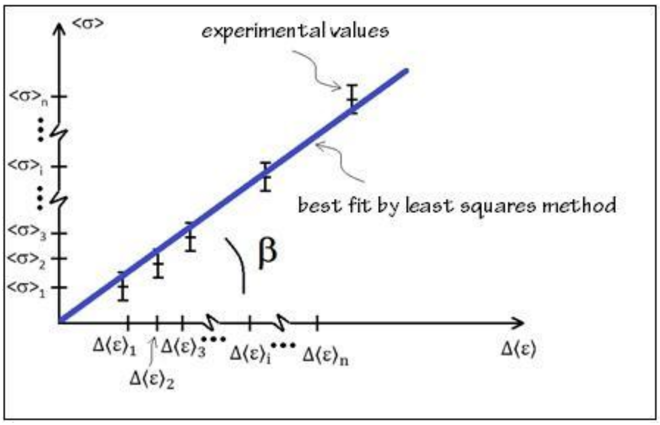
The mean relative displacements of the photoelastic fringes $\Delta \langle \boldsymbol{\varepsilon} \rangle_{\mathbf{i}}$, are obtained as follows:

$$\Delta \langle \varepsilon \rangle_{\mathrm{i}} = \frac{1}{\mathrm{m\,p}} \sum_{\mathrm{j}=1}^{\mathrm{m}} \sum_{\mathrm{k}=1}^{\mathrm{p}} \left(\frac{\mathrm{d}_{\|\,\mathrm{k\,j}}}{\mathrm{d}_{\mathrm{ref}\,\|}} - \frac{\mathrm{d}_{\perp\,\mathrm{k\,j}}}{\mathrm{d}_{\mathrm{ref}\,\perp}} \right), \quad (4)$$

$\mathbf{j}\,[\mathbf{1},\ \mathbf{m}]$ e $\mathbf{k}\,[\mathbf{1},\ \mathbf{p}]$, $\mathbf{d}_{\|\,\mathbf{k\,j}}$ e $\mathbf{d}_{\perp\,\mathbf{k\,j}}$ are the mean longitudinal and transverse distances, respectively, and $\mathbf{d}_{\mathbf{ref}}'\mathbf{s}$ are the reference distances.

The experimental values of $<\boldsymbol{\sigma}>_{\mathbf{i}}$ and $\Delta \langle \boldsymbol{\varepsilon} \rangle_{\mathbf{i}}$ with adjustment by the least squares method (VUOLO, 2003), will produce the graph outlined in **Figure 3**.

Figure 3 – Graph scheme with experimental values of mean stresses versus average relative displacements and best fit by least squares method.



Source: Own authorship.

From the graph, the slope is obtained, such that:

$$\langle \sigma \rangle = \mathrm{tg}(\beta)\, \Delta\langle \varepsilon \rangle \tag{5}$$

with,

$$\mathrm{tg}(\beta) = \frac{\langle \alpha_C \rangle}{C} \Rightarrow \langle \alpha_C \rangle = C\, \mathrm{tg}(\beta) \tag{6}$$

$\boldsymbol{tg(\beta)}$ is obtained directly from linear regression and C from typical values in the literature. The uncertainty of the equation (6) is, by error propagation method, described in Vuolo (2003) and Macwilliams and Sloane (1977),

$$\sigma_{\langle \alpha_C \rangle} = \langle \alpha_C \rangle \sqrt{\left(\frac{\sigma_C}{C}\right)^2 + \left(\frac{\sigma_{\mathrm{tg}(\beta)}}{\mathrm{tg}(\beta)}\right)^2} \tag{7}$$

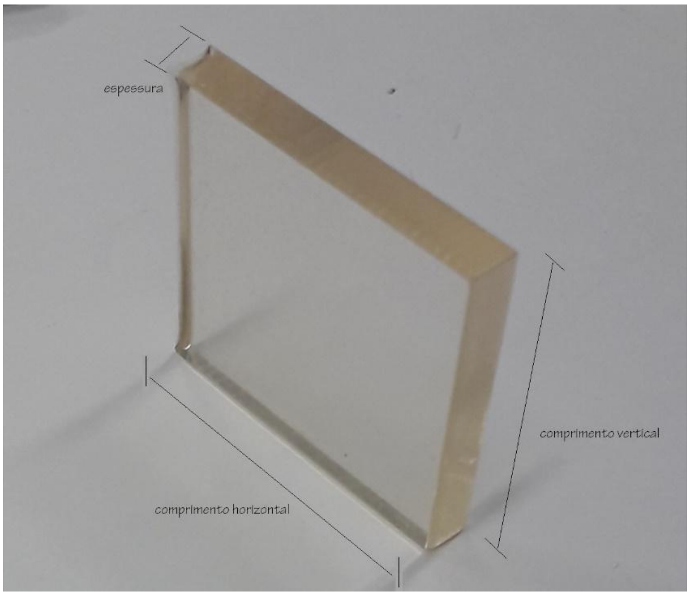
$\langle \boldsymbol{\alpha_C} \rangle$ is used as the system calibration coefficient.

METHODOLOGY

Epoxy resin produced by a polymerization process was used to make the photoelastic sample (DA SILVA et. al, 2015). This resin offers excellent adhesion to a large amount of materials and exhibits optical fringes when subjected to external stresses.

Figure 4 shows a photograph of photoelastic sample used in the experiments.

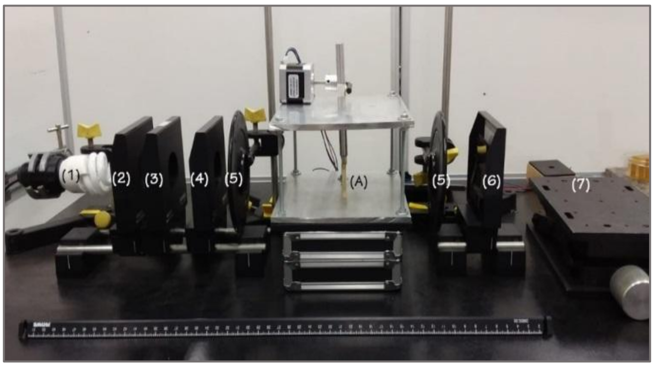
Figure 4 - Photoelastic sample



Source: Own authorship.

Figure 5 shows a photograph of the linear transmission polariscope used to perform data collection, with an automatic charging device coupled to the setup.

Figure 5 – Photograph of the photoelastic sample compression device configured in the linear transmission polariscope. (1) white light source; (2) filters and lenses; (3) color filter; (4) horizontal linear polarizer; (A) photoelastic sample; (5) 1/4 wave blades; (6) vertical linear polarizer; (7) base for digital camera



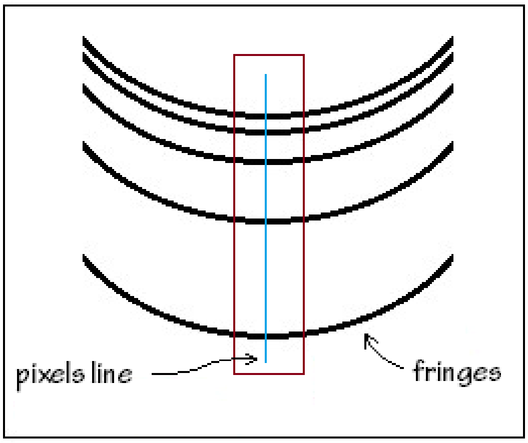
Source: Own authorship.

The spherical wavefront, produced by a white light source (1), is altered by the neutral density filter and the lens (2), making it approximately flat. Then, a color filter, (3), only allows light of a certain wavelength to pass through, so that only fringes of the same color are observed to facilitate the determination of the distances between the fringes. Two linear polarizers, (4) and (6), with orthogonal polarization states prevent, a priori, the passage of light. With the stresses of the device, the polarization state of the light changes as it passes through the sample (A), allowing the digital camera (7) to observe the isochromatic fringes produced. Quarter wave plates, (5) were added before and after the sample to eliminate most of the isoclinic (dark) fringes and allow for a purer image of the isochromatic fringes (of the chosen color).

With the aid of the loading device in **Figure 5**, automatic loading sequences were performed on the photoelastic sample in Figure 4 and a video was recorded with the digital camera. M videos were obtained each containing Q frames. The proposed method used computational analysis associated with error theory treatment (VUOLO, 2003; MACWILLIAMS; SLOANE, 1977). $\boldsymbol{Q}$ frames of $\boldsymbol{P}$ produced by a video were chosen from the consecutive compression stresses on the photoelastic sample. $\boldsymbol{G}$ groups, each containing $\boldsymbol{Q}/\boldsymbol{G}$ image frames, were separated for data processing. The mean mechanical elasticity modulus was determined by the method presented:

"… the external stresses, applied to the photeleastic sample, longitudinal were used. The mean deformations can be calculated from values obtained by lines of pixels along a chosen direction of the static fringes image, outlined in **Figure 6**, caused by the stresses applied to the photoelastic sample.

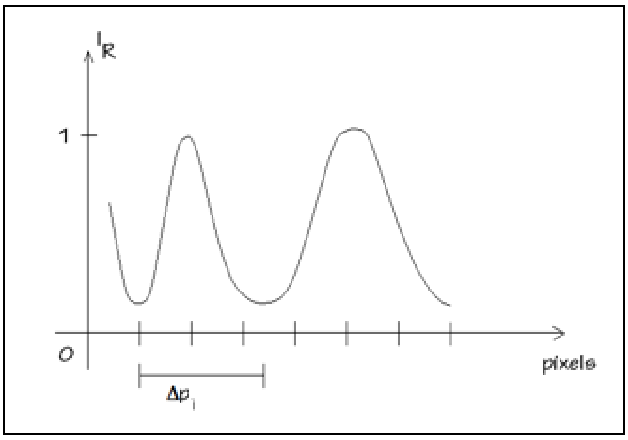
Figure 6: Line of pixels to determine the longitudinal mean deformation.



Source: Prado et al. (2020).

Each pixel line, shown in **Figure 6**, is associated with a relative intensity I_R, according to diagram in **Figure 7**.

Figure 7: Representation of the I_R curve versus ordered pixels.



Source: Prado et al. (2020).

The i-th relative intensity is determined from the equation,

$$I_{R_i} = \frac{I_{max} - I_i}{I_{max}} \quad (8)$$

Where I_i represents the i-th intensity, for the i-th pixel, with I = 1, 2, …, n. I_{max} represents the maximum intensity of the selected pixel line. The mean distance, <Dd>, between fringes is determined by the equation,

$$\langle \Delta d \rangle = \frac{1}{n}\sum_{i=1}^{n} \Delta p_i, \quad (9)$$

where Dp_i represents the difference between the positions of pixels in consecutive valleys or peaks, along the line of pixels. To determine the mean strain, <e>, of each image, the following equation is used,

$$\langle \mathcal{E} \rangle = \frac{d_{reference} - \langle \Delta d \rangle}{d_{reference}}, \quad (10)$$

where $d_{reference}$ is the reference distance, chosen accordingly, depending on the selected image. To determine the elasticity modulus, E, of the photoelastic sample, Hooke's Law represented in Equation (2) is used, such that,

$$\langle \sigma \rangle_j = E \cdot \langle \mathcal{E} \rangle_j \quad (11)$$

where $\langle \sigma \rangle_j$ is the j-th external mean stress and $\langle \mathcal{E} \rangle_j$ is the j-th mean strain, on the photoelastic sample, with j = 1, 2, …, m. From Equation (11), a graph similar to the schema in Figure 3 can be obtained…" (PRADO et. al, 2020).

With the mean mechanical elasticity modulus, represented by $\boldsymbol{tg(\beta)}$, **Figure 3**, and typical values in the literature of Da Silva (2017), for the optical dispersion coefficient, C**,** the dispersion factor, $\langle \boldsymbol{\alpha_C} \rangle$, was determined, through equation (6), as well as its uncertainty, through the error propagation method portrayed in the works of Vuolo (2003) and Macwilliams and Sloane (1977), equation (7).

RESULTS

A rectangular sample, with dimensions shows in **Table 1**, was made for the experiment in question.

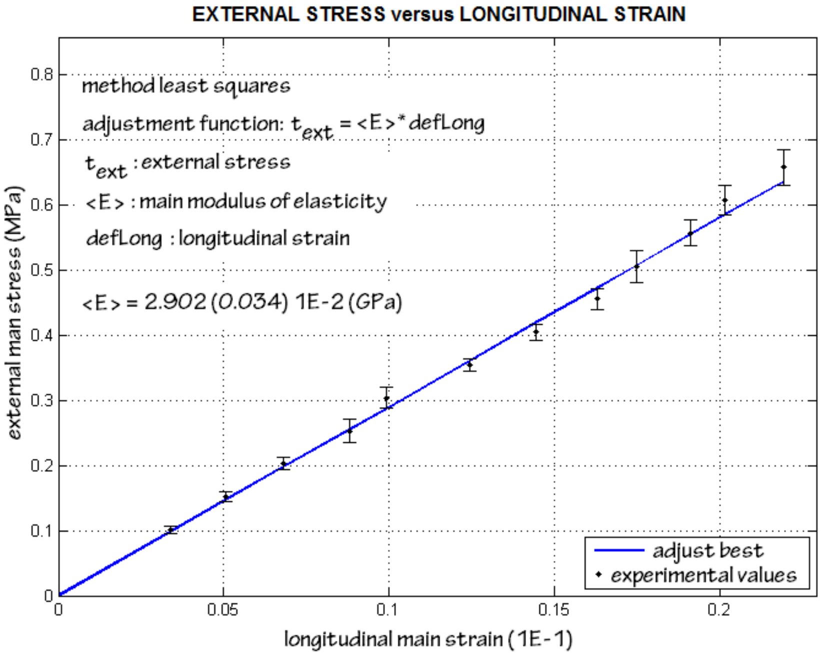
Table 1 – Geometric parameters of the photoelastic sample

	Dimensions (cm)
horizontal length	3,165±0,008
vertical length	3,022±0,006
thickness	0,474±0,005

Source: Own authorship.

The video obtained by the digital camera produced $\boldsymbol{P}$ = 4096 frames of images, from which $\boldsymbol{Q}$ = 4080 were selected, divided into $\boldsymbol{G}$ = 12 groups of $\boldsymbol{Q}/\boldsymbol{G}$ = 340 frames of images. Images have been converted to default containing 256 shades of grey, 8-bit. The graph in **Figure 8** shows the relationships between mean external stresses versus mean longitudinal strains for the photoelastic sample utilized.

Figure 8 – Mean external stresses versus mean longitudinal strains for the photoelastic sample



Source: Own authorship.

From the value $\mathbf{tg(\beta)} \equiv < E >= (2{,}902 \pm 0{,}034) \text{ x } 10^{-2}\text{GPa}$ and the mean typical value of the mean optical dispersion coefficient obtained in the literature, Da Silva (2017), $< C >= (2{,}79 \pm 0{,}13) \text{ x } 10^{-12}\text{m}^2/\text{N}$, the mean dispersion factor was obtained, $< \boldsymbol{\alpha}_\mathbf{C} >$ by equation (6), white your respective uncertainty, $\boldsymbol{\sigma}_{\langle \alpha_C \rangle}$, by equation (7), such that:

$$< \boldsymbol{\alpha}_\mathbf{C} > = (8{,}90 \pm 0{,}42) \text{ x } 10^{-5},$$

$< \boldsymbol{\alpha}_\mathbf{C} >$ is dimensionless, since the dimension of the modulus of elasticity, $[\boldsymbol{E}]$, is the inverse of the dimension of the optical dispersion coefficient, $[\boldsymbol{C}]$.

CONCLUSIONS AND PERSPECTIVES

The method proved to be efficient in determining the mean dispersion factor, $< \boldsymbol{\alpha}_{\mathbf{C}} >$, and, with it in hand, it is possible to use the same methodology to directly determine the optical dispersion coefficient of photoelastic samples. For this, just make samples from the same batch and with similar dimensions, then choose a test sample to determine $< \boldsymbol{\alpha}_{\mathbf{C}} >$ and use this coefficient to directly determine C. Thus, it is possible to characterize the photoelastic samples, determining more accurately their typical values, both for the modulus of elasticity, $\boldsymbol{E}$, and for their Poisson coefficients, $\boldsymbol{\nu}$. Another possibility is to indirectly determine values of $\boldsymbol{E}$ and $\boldsymbol{\nu}$ other materials inserted in photoelastic samples.

The perspectives from this work are: (i) to characterize several photoelastic samples, determining their values of C; (ii) characterize several photoelastic samples, determining their values of E and ν; (iii) apply the method presented in data obtained by the reflection photoelasticity technique, to other materials (example: metals) covered with thin layers of photoelastic resins; (iv) characterize other materials, determining values of C; (v) characterize other materials, determining their values of $\boldsymbol{E}$ and ν etc.

Determinação do fator de dispersão médio através da fotoelasticidade

RESUMO

Este trabalho se propôs a determinar o fator de dispersão médio, $<\boldsymbol{\alpha_C}>$, que está relacionado ao coeficiente de dispersão fotoelástica relativa de um material fotoelástico, submetido a tensões externas consecutivas, por meio de um método computacional direto associado à teoria de erros. Uma característica importante da fotoelasticidade, relacionada ao comprimento de onda da luz, é o coeficiente de dispersão fotoelástica, que é determinado em métodos tradicionais por processos estatísticos indiretos, pois depende de índices de refração, que são difíceis de determinar em materiais fotoelásticos. Portanto, encontrar formas alternativas para determinar este coeficiente é necessário e pode auxiliar nos processos que envolvem o uso da fotoelasticidade. O valor médio do fator de dispersão obtido foi $<\boldsymbol{\alpha_C}> = (8{,}90 \pm 0{,}42) \times 10^{-5}$, que funciona como um calibrador do sistema para permitir a determinação do coeficiente de dispersão fotoelástica de materiais fotoelásticos por fotoelasticidade, com maior precisão.

PALAVRAS-CHAVE: fotoelasticidade, materiais fotoelásticos, birrefringência, teoria de erros.

Determinación del factor de dispersión medio utilizando fotoelasticidad

RESUMEN

Este trabajo propuso determinar el factor de dispersión medio, $< \boldsymbol{\alpha_C} >$, el cual está relacionado con el coeficiente de dispersión fotoelástica relativa de un material fotoelástico, sometido a esfuerzos externos consecutivos, mediante un método computacional directo asociado a la teoría de errores. Una característica importante de la fotoelasticidad, relacionada con la longitud de onda de la luz, es el coeficiente de dispersión fotoelástica, que se determina en métodos tradicionales mediante procesos estadísticos indirectos, ya que depende de índices de refracción, que son difíciles de determinar en materiales fotoelásticos. Por lo tanto, encontrar formas alternativas para determinar este coeficiente es necesario y puede ayudar en los procesos que involucran el uso de fotoelasticidad. El valor medio del factor de dispersión obtenido fue $< \boldsymbol{\alpha_C} > = (8{,}90 \pm 0{,}42) \times 10^{-5}$, el cual funciona como calibrador del sistema para permitir la determinación del coeficiente de dispersión fotoelástica de materiales fotoelásticos por fotoelasticidad, con mayor precisión.

PALABRAS CLAVE: fotoelasticidad; materiales fotoelásticos; birrefringencia; teoría de errores.

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