RBECT Revista Brasileira de Ensino de Ciência e Tecnologia

Revista Brasileira de Ensino de Ciência e Tecnologia
ISSN: 1982-873X

https://periodicos.utfpr.edu.br/rbect

The Notions of 'Reality' in Mathematical Modeling Research in Mathematics Education: A Systematic Review Supported by IRaMuTeQ Software

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ABSTRACT

The objective of this study was to gather research through a systematic review to reveal how the phenomenon of 'reality' is portrayed in MM research in ME, and to identify the notions of 'reality' based on MM studies in ME. To achieve this, a qualitative study was conducted, guided by a Systematic Review of literature relevant to the field of Mathematical Modeling in Mathematics Education. Searches were carried out in seven sources, including databases, portals, virtual libraries, conference proceedings in the field, and citation searches, namely: the CAPES Theses and Dissertations Catalog and Journals Portal, Scielo, Eric Research, Google Scholar, Google, and the National Conference on Mathematical Modeling – CNMEM. The Systematic Review followed a protocol based on the POT strategy (Population, Outcome, and Types of studies), adapted for use in qualitative research. Mendeley was used to organize, manage references, and remove duplicates from the studies selected for the review. Discursive Textual Analysis – DTA, supported by the IRaMuTeQ software, was partially used to analyze the textual corpus. As a result of this analysis, three categories emerged: 1) Reality: its objectives and characteristics in the context of Mathematical Modeling; 2) Reality as a guiding element of Mathematical Modeling: its dimensions and epistemological aspects; 3) Reality seen as an interdisciplinary thematic unit and the teacher's role in selecting themes for Mathematical Modeling activities. The study's findings revealed that notions of reality in the context of Mathematical Modeling are plural. Evidences were found of certain philosophical-epistemological roots of reality, such as Platonism, including sensory world reality or Platonic reality, and the idea of realism. Based on these findings and through immersion in the literature, three dimensions of reality present in Mathematical Modeling conceptions were identified: initial reality (given reality), intermediate reality (reality to be modeled), and the mathematical model (pseudo-reality).

KEYWORDS: Mathematics Education. Mathematical Modeling. Reality. Systematic Review. IRaMuTeQ.

1 INTRODUCTION

Reality is considered in the literature (BARBOSA, 2001; ARAÚJO, 2002; ALMEIDA; DIAS, 2004; BURAK, 2004; NEGRELLI, 2008; BASSANEZI, 2009; KLÜBER, 2012) as a feature that appears in all conceptions of Mathematical Modeling (MM)[1], where inferences, debates, and problematizations are produced. In this sense, Barbosa (2004) states that it is necessary to understand what MM is, why to do it, and how to do it, delving into practical and epistemological issues of this field of study. Thus, Barbosa emphasizes the importance of teachers understanding the different ways of practicing MM from the perspective of Mathematics Education (ME), which is often generically conceptualized as the application of mathematics to other areas of knowledge, representing a theoretical delimitation. While it may not be necessary to impose strict boundaries on MM, it cannot be a broad 'umbrella' under which almost anything fits: clarity is required regarding what is called MM. Barbosa (2004, p. 01) asserts that "other times, the parameters of Applied Mathematics, expressed in explanatory schemes [...] are borrowed to define MM."

MM is understood as one of the means to question reality. For this purpose, a critical view of the demands of MM proposals in the classroom is necessary. To discuss MM epistemologically in ME is to position oneself in a research field that is inherently challenging, as Bueno (2011) considers that many conceptions exist within this research area.

Klüber (2012) emphasizes that it is from these multiple meanings attributed to MM in ME, viewed here as a phenomenon, that conducting an investigation from a phenomenological perspective becomes pertinent: "It is in this tangle of meanings that a phenomenological investigation can show its value, clarifying aspects of the MM phenomenon in ME, confirming certain aspects, refuting others, and still SHOWING its complexity" (KLÜBER, 2012, p. 39).

Faced with this initial theoretical challenge, the debates presented here do not intend to exhaust the theme of reality but rather to understand it as an event or phenomenon, analyzed through DTA (Discursive Textual Analysis) in this study. This involves bringing evidence and characteristics to understand how reality manifests in the context of MM from the perspective of ME.

Klüber (2012) highlights the need to explore and reflect on the diversity of debates in the MM field regarding the pedagogical intentionality of such activities. The author also emphasizes the importance of understanding reality in the context of MM from a philosophical-epistemological perspective, addressing relevant concerns about the state of knowledge on the subject:

> This attitude allows overcoming the current state of knowledge in a given area and, in a more philosophical sense, a region of inquiry, and more specifically on the phenomenon in question: Mathematical Modeling in Mathematics Education. It rejects a more naive view of the factual given in the production of Mathematical Modeling in Mathematics Education, that is, on superficial understandings of important themes such as reality (KLÜBER, 2012, p. 51).

Negrelli (2008) also provides theoretical contributions regarding reality, affirming the evident need for studies aimed at investigating the epistemological issues of the guiding elements of MM. This is particularly relevant as much of the research conducted in Brazil and other countries on MM in ME focuses on practical investigations within the classroom.

Given the evident need for studies valuing an epistemological debate on the guiding elements of MM proposals in ME—particularly concerning reality—this research arises. Its general objective is to analyze studies gathered through a systematic review to reveal how the phenomenon of ‘reality’ is portrayed in MM research in ME and to identify notions of ‘reality’ based on MM studies in ME. The specific objectives are: 1) To highlight the nature of the discourses on reality mobilized in MM research in ME, adopting the IRaMuTeQ software for partial data processing associated with DTA; 2) To discuss and analyze the possible dimensions of reality guiding MM proposals in ME.

2 THEORETICAL-METHODOLOGICAL FRAMEWORK OF THE RESEARCH

2.1 Nature of the Research

This study encompasses a literature review concerning the field of MM in ME. It investigates the notions of the term 'reality' in MM proposals, based on the following question: **How does 'reality' manifest in MM research in ME?**

To address this research question, albeit provisionally, a qualitative research approach is adopted. According to Lüdke and André (1986), qualitative research also aims to gather and confront evidence about a given body of knowledge. In this way, it demonstrates its social character, as it arises from the curiosity, intelligence, and investigative activity of individuals, based on what has already been produced and systematized and what remains to be explored.

This study is also anchored in the perspective of Discursive Textual Analysis (DTA) as outlined by Moraes and Galiazzi (2006), aiming to reveal how the phenomenon manifests. This choice is believed to align with the objectives of this study. By employing the PRISMA protocol guided by the Systematic Review methodology, DTA as a data analysis methodology, and the IRaMuTeQ software as an indispensable tool, this study effectively organizes, describes, deconstructs, interprets, and analyzes the corpus, thereby clarifying evidence about the phenomenon under investigation.

What is a systematic literature review? According to the Cochrane Glossary (2022), a systematic review is a structured summary of research results addressing a specific research question. It employs systematic and explicit methods to identify, select, and critically evaluate research, followed by collecting and analyzing studies included in the review.

The first step of a systematic review involves developing a research protocol. In this case, the POT strategy (Population, Outcome, Type of Research) was employed in the research protocol, providing precision and rigor for data collection. It is important to highlight that the focus is not on identifying the effect of an intervention or exposure but on summarizing and analyzing existing studies on reality in the context of MM. In addition to the POT strategy, the PRISMA protocol was used, as this systematic review involves a significant number of studies. PRISMA was adopted to facilitate transparent and specific reporting of these studies.

The chosen search terms were: Reality, Mathematical Modeling, and Mathematics Education. To ensure a more targeted search, the terms "Reality"[2], "Mathematical Modeling", and "Mathematics Education" were enclosed in quotation marks and expressed in both Portuguese and English, considering the scope and interdisciplinary nature of MM in dialogue with other fields. Boolean operators OR and AND were employed, resulting in the following search expression: "Mathematical Modeling" OR "Modelagem Matemática" AND "Reality" OR "Realidade" AND "Mathematics Education" OR "Mathematics" OR "Educação Matemática". This approach refined the search, ensuring a more precise and objective exploration, as illustrated in Table 01:

Table 01 - Search Terms Used in the Research

Databases	Keywords	Search terms
CAPES/CAF-e	Modelagem Matemática/Realidade	"Modelagem Matemática", "Realidade" e "Educação Matemática"
CAPES - CATÁLOGO DE TESES E DISSERTAÇÕES	Modelagem Matemática/Realidade	"*Mathematical Modeling*" *OR* "Modelagem Matemática" *AND* "*Reality*" *OR* "Realidade" *AND* "*Mathematics Education*" *OR* "*Mathematics*" *OR* "Educação Matemática"
ERIC	Modelagem Matemática/Realidade	"*Mathematical Modeling*" *OR* "Modelagem Matemática" *AND* "*Reality*" *OR* "Realidade" *AND* "*Mathematics Education*" *OR* "*Mathematics*" *OR* "Educação Matemática"
SCIELO	Modelagem Matemática/Realidade	"*Mathematical Modeling*" *OR* "Modelagem Matemática" *AND* "*Reality*" *OR* "Realidade" *AND* "*Mathematics Education*" *OR* "*Mathematics*" *OR* "Educação Matemática"
CNMEM	Modelagem Matemática/Realidade	"Modelagem Matemática", "Realidade"
GOOGLE	Modelagem Matemática/Realidade	"Modelagem Matemática", "Realidade"
GOOGLE ACADÊMICO	Modelagem Matemática/Realidade	"Modelagem Matemática", "Realidade"
OUTRAS FONTES	Modelagem Matemática/Realidade	"Modelagem Matemática", "Realidade"

Source: Of the text (2022).

2.2 Types of Studies to Be Included

Studies focusing on the field of MM research from the perspective of ME, which highlight a discourse on reality within the context of MM in ME, can be qualitative or quantitative in nature. These studies may be developed in Brazil or abroad, within ME, and may be bibliographic or empirical in context.

2.3 Inclusion and Exclusion Criteria

In this protocol, the inclusion criteria consider research that includes the term Mathematical Modeling in Mathematics Education and the term reality in the title, abstract, or keywords. These may include qualitative studies in the form of articles, books, conference proceedings, theses, dissertations, or any scientific document addressing MM from the perspective of ME, whether bibliographic or empirical in nature. Articles from peer-reviewed and non-peer-reviewed journals that provide full-text access and are freely available in the databases searched and in the gray literature are also considered. Articles identified via citation and email searches may also be included if they are highly relevant to the research (e.g., sentinel articles).

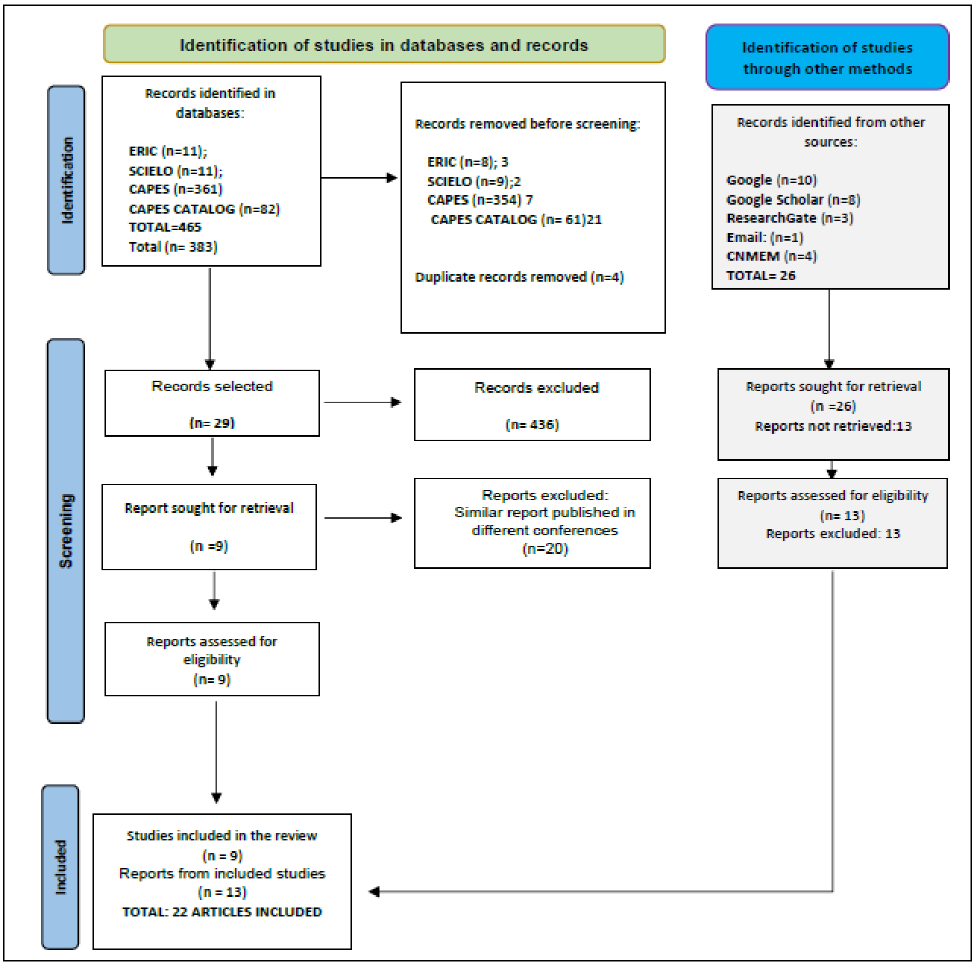
Research that does not feature the term Mathematical Modeling from the perspective of ME and the term reality in the title, abstract, or keywords will not be considered. Studies on MM that do not specifically involve the field of ME will also be excluded, as well as articles not available in full text or freely accessible in the consulted databases. After the search phase, Mendeley was used as a resource to organize and manage references and remove duplicates from the studies selected for the systematic review.

3 RESEARCH RESULTS

This section highlights the results of the textual corpus analysis, partially analyzed using IRaMuTeQ. The textual corpus comprises 22 texts resulting from the systematic review, including 16 articles, 2 essays, 2 dissertations, and 2 theses, as shown in Figure 1.

A total of 22 studies were included in the research, organized, and encoded into a single file. After preparation, the processing was carried out using the IRaMuTeQ software, which generated the following general information: 535 text segments (TSs), a total of 21,877 word occurrences, with 2,219 distinct forms, 2,338 lemmas, 2,219 active forms, and 129 supplementary forms. The number of active forms with a frequency of three or more was recorded, and the average number of forms per segment was 40.891589. Five classes were identified, with a utilization rate exceeding 75% (80.19%). The total processing time was 0 hours, 0 minutes, and 36 seconds.

Figure 1 – PRISMA Flowchart

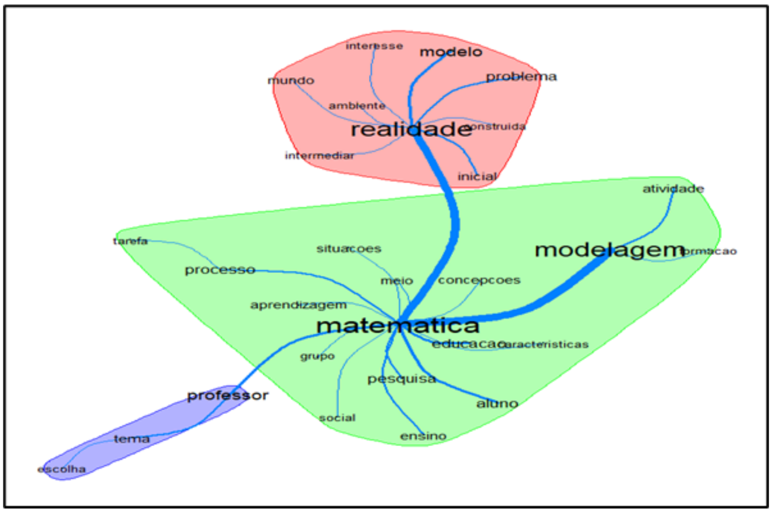


Source: Adapted from Page *et al.* (2021).

Another way to analyze the data chosen was the similarity analysis (Figure 2), which highlights, through statistical indicators, the connections between words in a textual corpus. According to Salviati (2017), similarity analysis is a branch of mathematics based on graph theory, which deals with the relationships between objects in a set, objectively identifying the occurrences between words.

Revista Brasileira de Ensino de Ciência e Tecnologia
ISSN: 1982-873X

Figure 2 – Similarity Analysis Graph resulting from IRaMuTeQ processing



Source: Research data (2023).

The processing of the corpus in IRaMuTeQ resulted in this similarity tree, represented through a graph (graphot) with 29 words. The co-occurrence and connection between lexical forms are highlighted in three communities, marked in green, coral, and blue, distributed as follows: **Green Community**: mathematics (499), modeling (299), student (101), process (93), research (75), education (70), activity (64), teaching (55), conceptions (43), medium (34), situations (33), learning (29), social (28), task (27), group (27), training (17), and characteristics (16). **Coral Community**: reality (422), model (123), problem (73), initial (56), world (36), intermediate (21), environment (18), interest (18), and constructed (3). **Blue Community**: teacher (114), theme (79), choice (24).

3.1 Unitarization process under the perspective of ATD and IRaMuTeQ

Magno and Gonçalves (2023) were the authors referenced to systematize the meaning units for text segments (IRaMuTeQ). They understand that the systematization of text segments (STs) is carried out by ATD, through an inventory of information, to create meaning units that justify the emergence of categories (Figure 3).

Figure 3 – Infographic of the organization of text segments (STs) using ATD

Source: Magno and Gonçalves (2023).

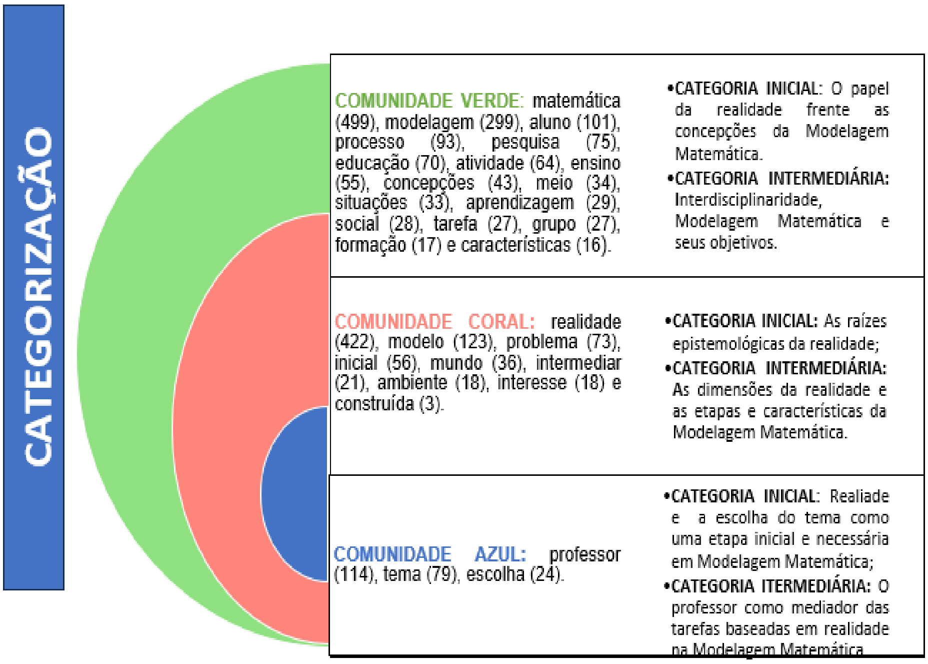
In the unitarization process of the text segments resulting from the processing of the textual corpus, it is noteworthy that the titles of theses, dissertations, essays, and articles that generated the corpus were subsequently coded (Example of coding for a thesis: ST01THESIS01 – ST01 – Text Segment 01 + Thesis 01 = the type of study and its location in the corpus). This structure generated the identification code composition.

3.2 Categorization

The initial and intermediate categories that emerged are elucidated below (Figure 4), based on a process of construction and reconstruction of interpretative movements regarding how reality manifests in the MM process.

Thus, the research movements described in the infographic above led, through a constant process of immersion and recursion, to the selection of the following final categories: **Green Community**: Reality, its objectives, and characteristics within the scope of MM. **Coral Community**: Reality as one of the guiding elements of MM, its dimensions, and epistemological aspects. **Blue Community**: Reality seen as an interdisciplinary thematic unit and the role of the teacher in choosing the theme for MM activities.

Figure 4 – Infographic of the research categorization process



Source: Author's work (2023).

4 COMMUNICATION OF RESULTS

This section discusses the final categories, which serve as the basis for the metatexts below and, consequently, for understanding the phenomenon under study. Regarding the structure, it will follow this format: Metatext 01 addresses the first final category related to the green community (reality: its objectives and characteristics within the scope of MM); the second metatext presents inferences on the second final category of the coral community (reality as one of the guiding elements of MM in the epistemological aspect and its dimensions); and the third and last final category constitutes the discussions of the third metatext (reality seen as an interdisciplinary thematic unit and the role of the teacher and the student in choosing the theme in MM activities).

4.1 Metatext 01 – Reality: its objectives and characteristics within the scope of MM

Would one of the objectives of MM be to demonstrate the applications of mathematics through the contextualization of reality? Or, could it be said that reality provides support for teaching mathematics through MM? Burak (2004) argues that, during exploratory research, one of the stages of MM, students seek the necessary resources for addressing the proposed theme in reality.

Examining the excerpt ST01A08, it is possible to highlight one of MM's objectives: solving tasks based on reality, where resources are sought in reality to justify the presence of mathematics.

> Solving tasks based on reality is an important objective in mathematics education and is anchored in educational standards determined by mathematical modeling skills. These tasks require a thorough examination of the real world and an understanding of the text to solve them successfully (ST01A08).

Beltrão (2009) emphasizes that there are at least two theoretical concepts concerning MM that are inherent to curriculum-oriented objectives. These concepts address questions related to reality, applications, contextualization, and mathematics: MM as a vehicle and MM as content. In MM as a vehicle, one of its main characteristics is that real-world problems are used to motivate and provide resources that can lead to exploring certain mathematical content. In this case, the needs of the mathematics curriculum dictate the choice of problems to be studied with students. In MM as content, the goal is to enhance students' ability to solve problems based on the external world and evaluate the quality of their solutions.

Beltrão and Igliori (2010) highlight that through historical studies and research on the topic of Applications and Modeling, it is possible to understand some epistemological principles of these approaches. One of these principles is that mathematics plays an important role in explaining real-world phenomena. To achieve this goal, Mathematical Modeling (MM) provides a conducive learning environment. According to Beltrão and Igliori (2010), the objectives and characteristics of MM, in relation to "explaining the reality phenomena," can be viewed in two ways:

> Thus, we assume the term "Application" as an action of Mathematics directed toward reality. It is as if we were asking: "Knowing certain mathematical topic(s), where can we apply them?" The term "Modeling" portrays a different type of action, starting from reality and moving toward Mathematics. It is as if we were asking: "Where can I find some Mathematics to help us address this problem?" In other words, Modeling enables understanding or solving problems in specific segments of the real world. In this case, solving a problem involves collecting real data, which provides information about the situation of interest (BELTRÃO; IGLIORI, 2010, p. 19).

In attempting to understand the phenomenon, MM crosses into other fields of knowledge beyond mathematics, as evidenced in excerpt (ST03A09): “[...] it allows the teacher to explore mathematical concepts in contexts other than mathematics itself and enables students to observe some of its applications.”

Interdisciplinarity emerges as a central element within the discussion of reality in the MM process. By seeking resources in reality to understand phenomena, discussions can follow multiple routes and "enhance the perspective of the islands provided by the disciplines," as stated by Beltrão and Igliori (2010, p. 32).

Some objectives and characteristics of reality within the scope of Mathematical Modeling (MM), as evidenced in the analyzed studies, are revealed through various stages. For instance, the "choice of topic," which seeks to address tasks referencing reality (selecting a topic related to some aspect of reality to solve), as highlighted by Bassanezi (2009). In the "exploratory research" phase, according to Burak (2004), theoretical support or field research is pursued—essentially, reality is investigated to justify the presence of mathematics.

4.2 Metatext 02 – Reality as a Guiding Element of MM: Its Dimensions and Epistemological Aspects

Starting from the premise that reality is one of the guiding principles of MM activities, reality is discussed from an epistemological perspective. According to Negrelli (2008, p. 14), reality is conceived as a fundamental factor in conducting activities and is present in all MM approaches, as it is "a characteristic element that, in one way or another, takes reality as the starting point of the MM process."

Negrelli (2008, p. 22, emphasis added) also views reality as a guiding component in MM proposals in ME: "a characteristic element present in any MM perspective: the reference to a reality to be modeled." This emphasized expression represents one of the initial dimensions emerging in the MM process, which will be further discussed. Araújo (2002) supports this assertion by elucidating the presence of reality in MM proposals, even with differing objectives:

> Mathematical Modeling, regardless of the context in which it is applied, aims to solve some real-world problem using mathematical theories and concepts. Differences arise as objectives are defined—what problem is to be solved, which reality it belongs to, and how mathematics is conceptualized and relates to that reality [...] (ARAÚJO, 2002, p. 20).

Bassanezi (2009, p. 25) emphasizes the necessity of translating "a problem from a particular reality into mathematics, where it is analyzed, and through an inverse interpretation, results are obtained in the original problem's language." This statement suggests at least two understandings of reality in MM: the existence of multiple dimensions of reality, as mathematical resources are drawn from a particular reality, and the notion that mathematics itself is not considered part of reality.

It becomes evident, however, that MM perspectives share a commonality: the objective of addressing a real-world problem or situation. Thus, it is important to characterize what constitutes 'reality' or a 'reality problem' and how reality guides this practice. In this regard, Negrelli (2008) identifies at least three dimensions of reality in the MM process:

> The construction of an intermediate reality situated between the initially focused reality and the mathematical model to be developed is a

> fundamental stage. However, beyond these three stages—the initial reality, the intermediate reality, and the mathematical model (ST02TESE01).

Negrelli (2008) considers that the reality providing resources for MM activities, referred to as the initial reality, encompasses elements of economic, social, and physical nature, among others. In this process, the modeler translates a problem from this reality into MM, creating another dimension of reality: the intermediate reality. According to Negrelli (2008), during the interaction phase, students analyze and simplify the initial situation (initial reality) to construct an intermediate reality.

Thus, the author highlights three dimensions of reality in the MM process: the initial reality (the given reality), the intermediate reality (the constructed reality to be modeled), and the model. In this process, it can be said that the intermediate reality holds a higher status as reality than the model. The author characterizes this dimension as a subset of the initial reality:

> But where does the 'problem' (sic) to be translated into mathematics reside? In reality? We believe not. There is an intermediate moment between reality and the model in the MM process, consisting of problematization that implies another reality, which we will call intermediate reality. This is not yet the model. It is a simplification of the initial reality, achieved through hypotheses and simplifications, from which the problem will be formulated (NEGRELLI, 2008, p. 34-35).

Given the characterization of reality in MM, within the scope of BE and considering its dimensions, attention is drawn to its epistemological roots. Reality, as conceived through the experiences and interests of students or teachers, is described as follows (ST02A12):

> According to Bicudo (2000), reality is constructed, perceived, and created. Reality is the world, but not the Cartesian world that exists independently of human beings. It is the world understood as a horizon of relationships in which we live and position ourselves with our students (ST02A12).

Realism, as a theoretical basis for notions of reality in MM, frequently appears in studies, as highlighted in ST01A14:

> For this study, definitions were selected characterizing MM as a means of describing and/or studying real-world problems. In all of them, the relationship between mathematics and reality aligns with the philosophical current of realism (ST01A14).

Negrelli (2008) asserts that one feature of MM is taking reality as its starting point but questions what constitutes this reality and the kind of reality MM addresses. These answers are rooted in realism:

> At first, we can understand that it deals with reality composed of economic, physical, social, political, psychological elements (sic), whose existence we can assume from a realist perspective. MM aims to translate a problem from this reality into mathematics to understand it through problem-solving (NEGRELLI, 2008, p. 34).

This notion of reality anchored in realism, according to Negrelli (2008), represents a simplified version of Kant's empirical realism, where reality's conception involves recognizing the existence of things independently of our knowledge of them. Moraes (2004) believes that this vision of realism generates an objective reality free from philosophical interpretation.

Another characteristic identified in research is Platonic reality, serving as the epistemological basis for reality notions in MM, as per ST01A15:

> The relationship between mathematics and reality: it was observed that the proposals also contain the essence of reality and problem-situations emerging from it [...] reality problems belong to what Plato called the sensory world (ST01A15).

The excerpt refers to Plato's Allegory of the Cave, which explains how **the reality of our knowledge** works. It figuratively illustrates that people are prisoners in a cave from birth, and the shadows cast on the wall constitute what they perceive as the real world.

Cifuentes and Negrelli (2012) suggest a third stage in MM, the pseudo-reality, considering mathematics as part of reality illuminated by Platonism, structuralism, and formalism. These philosophical underpinnings support the three stages of MM: the initial reality, the intermediate reality, and the pseudo-reality, engaging with three different dimensions of reality:

> We adapted the process to study situations from the Platonic mathematical world, considering them initial realities that can be mathematically modeled. This approach outlines a plural conception of mathematics, revealed in different forms during the modeling process. It is interpreted as Platonic in the initial reality, structuralist in pseudo-reality guided by mathematical intuition, and formalist in the model structured through axiomatic systems and logic. This process highlights mathematical experience as a means of accessing the initial reality and constructing pseudo-reality (CIFUENTES; NEGRELLI, 2012, p. 795).

Through a simplification process initiated in pseudo-reality, Cifuentes and Negrelli (2012) propose a new representation as a third stage, replacing natural language with mathematical language, deemed more suitable. This constitutes the model, where pseudo-reality functions as a lens, allowing us to understand the initial reality.

4.3 Metatext 03 – Reality as an Interdisciplinary Thematic Unit and the Role of Teachers and Students in Choosing a theme in MM

Negrelli (2008, p. 5) points out that Mathematical Education (ME) presents itself as a field of constant dialogue, partnership, and interaction among different areas of knowledge, such as philosophy, linguistics, epistemology, psychology, sociology, pedagogy, history, anthropology, among others, in a joint effort to seek increasingly adequate approaches to mathematics in the educational field.

In this regard, Negrelli (2008, p. 5) emphasizes that mathematics should support the other disciplines in such a way that it is necessary to: "take care not to let it (sic) become so diluted in studies that fit within the scope of ME, to the point where its identification becomes difficult or imperceptible, as its conscious consideration itself becomes doubtful." Thus, interdisciplinarity proves its value in ME as one of the central elements in teaching and learning processes, especially in today's society, where complexity is significant.

Regarding the characteristics of interdisciplinarity and its relationship with mathematics, Negrelli (2008, p. 5) states:

> Interdisciplinarity involves a differentiated way of thinking about the issues that arise. In it, elements traditionally recognized as belonging to a specific area foster attitudes that confer meanings that would not be possible through other means. The possibility of thinking historically or philosophically about mathematics, or mathematically about physical, artistic, or historical issues, is a manifestation of the interdisciplinary nature inherent in humanity's construction of knowledge.

Thus, Cifuentes and Negrelli (2012) conceive interdisciplinarity as a way of thinking and an attitude toward a given body of knowledge without any subordination. Based on this, Negrelli (2008) argues that ME cannot be considered a subfield of Mathematics, Education, or any other area, but rather as the result of an interdisciplinary effort.

David and Tomaz (2008) propose selecting themes to promote interdisciplinarity, as this can significantly enhance student engagement and contribute to the development of critical thinking skills (David & Tomaz, 2008, p. 20).

Burak (2004) emphasizes that the theme can be interdisciplinary, meaning it does not necessarily need to have an immediate connection with mathematics or mathematical content but must be of interest to the students. Blum *et al*. (2007, p. 8) view reality as an interdisciplinary field and support this idea when they highlight that: "we understand applications as a way to move from mathematics to the extramathematical world, and conversely, we understand Mathematical Modeling (MM) as a way to move from the real world to mathematics."

Regarding the approximations and simplifications of reality in the MM process, Bassanezi (2009, p. 24) asserts that MM is only effective “when we become aware that we are always working with approximations of reality, i.e., that we are dealing with representations of a system or part of it.” A common way to refer to reality or seek insights for MM activities is through themes, which are fragments of reality aligning with students' or teachers' interests, as suggested by Burak (2004).

Regarding students' interest in reality, Barbosa (2001) considers that in Brazil, MM in basic education is closely tied to the concept of projects. In this context, students work in groups to choose themes of interest to be investigated through mathematics, guided by the teacher. Theme selection is one of MM's steps. Burak (2004) proposes five stages in MM, guided by students' or groups' interests, and aligned with educational level needs: 1) theme selection; 2) exploratory research; 3) problem identification; 4) problem-solving and mathematical content development within the theme’s context; and 5) critical analysis of the solutions.

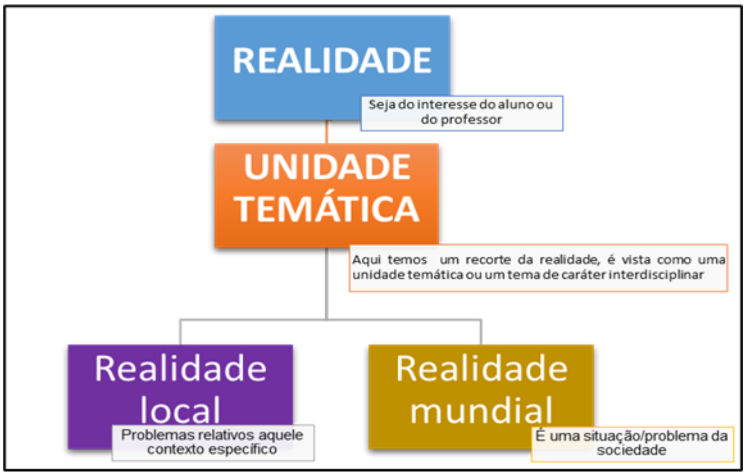
On theme selection (the first step), excerpts (ST01A13 and ST02A13) highlight that these thematic units can address global or local realities:

> Teachers justified theme selection based on the reality surrounding students, whether global or local realities of a given school community, linked to what teachers believe to be of interest to students (ST01A13).

> The theme chosen by teacher Cau is identified as a global theme, as 'recycling' is a societal issue. Conversely, teacher Márcia's chosen theme is identified as local and specific, "the school's collapsing wall" (ST02A13).

The following Figure 5 illustrates how reality is interpreted and approached in MM proposals, particularly regarding theme selection and interdisciplinarity. Through research analysis and methodological choices, it was evident that reality is viewed as a fact or phenomenon (a fragment, thematic unit, or theme) relevant to students' interests, appearing in studies as either local or global reality.

Figure 5 – Scheme for Interpreting Reality in Theme Selection in MM



Source: Research Author (2023).

Commonly, in studies on MM, reality is described as an idea, fact, or phenomenon related to the community where an individual is situated. This concept aligns with Moraes's (2004) perspective of "constructed reality," where reality is built by a community based on a consensus of its members' beliefs. Thus, rather than perceiving reality as an individual perspective, a broader notion emerges, one in which reality is collectively constructed by an entire community, not individually. In this sense, both local and global reality are understood as constructed realities.

The understanding of MM presented by Barbosa (2001) emphasizes real-life situations supported by relevant conditions, such as the following themes: the growth of a plant, school attendance rates, the construction of a sports court, advertising costs for a company, commercial turkey farming, and a water distribution system in a building. In this context, the author refers to an interest in real situations.

Barbosa (2004) situates and characterizes the notion of reality in MM within basic education, presenting the elements that shape MM activities: reality, mathematics, learning environment, and the nature of the activity. When referring to reality, the author uses the term: "situations with a reference to reality."

> [...] I can summarize by saying that Modeling, for me, is a learning environment in which students are invited to problematize and investigate, through mathematics, situations with a reference to reality. I have tried to clarify for myself what I understand by Modeling, considering the specificity of Mathematics Education. The reader will notice that I sought to define it in terms of the context in which it is developed (the school), the nature of the activity (investigation), and the domains it involves (mathematics and areas

> with a reference to reality). This understanding aims to delineate a specific region encompassing the activities I call Modeling” (BARBOSA, 2004, p. 3).

When it comes to selecting themes as a way to promote interdisciplinarity, MM emerges as a teaching methodology in mathematics that shares this initial approach to activity development. It is within this investigative cycle that reality appears as a thematic unit.

Burak and Klüber (2008) believe that there should be a balance between the teacher, student, and environment. In the proposed stages, tasks always unfold through constant interaction between teacher, student, and environment, without any single element dominating. The interaction among these three dimensions (teacher-environment-student) is key: the student must seek knowledge, the teacher must mediate, and the environment serves as the source from which information is gathered to support the research.

5 SOME CONSIDERATIONS

From the question "How is reality manifested in MM research within ME?", a decision was made to conduct a systematic literature review. The results of this search were organized in a digital library using Mendeley software. This tool was crucial for removing duplicates and organizing the studies found. IRaMuTeQ and ATD were essential for data processing and analysis.

By gathering the studies through the methodological approach mentioned earlier, it became possible to identify multiple references to reality, expressed in forms such as: common-sense reality, real world, real-world problems, everyday problems, students' reality, and others. Therefore, a study was necessary to seek answers, albeit provisional, that could characterize the references to the term "reality" in MM.

Through a process of immersion, description, and interpretation of the studies, initial, intermediate, and final categories emerged. The final categories were as follows: the first category ("Reality: its objectives and characteristics in the context of MM"), the second category ("Reality as one of the guiding elements of MM: its dimensions and epistemological aspects"), and the third category ("Reality seen as an interdisciplinary thematic unit and the teacher's role in selecting the theme and activities for MM").

Based on the methodological choices and studies that allowed analysis and inference about the phenomenon in such a way that it could reveal itself, it was possible, beginning with the analysis of the first category, to understand that one of the objectives of the phenomenon "reality" is to provide support for the development of MM activities and to demonstrate mathematical applications.

The second final category showed that reality has its epistemological roots in Platonism, which, according to Negrelli (2008), is a type of realism, as well as in structuralism and empiricism. From these aspects, its dimensions in the MM process were understood: the initial reality (the dimension of reality to be understood, the given reality, reality as it appears, or perceived reality); the intermediate reality (the reality to be modeled, mathematical reality, constructed reality); and the model (pseudo-reality).

Reality as a thematic unit emerged in the third category. Evidence from the units of meaning in the categorization process was discussed and presented. From this investigative structure, the following results were reached: reality appears as an interdisciplinary thematic unit. The reason for this is that MM itself draws on various sources of knowledge, including mathematics, to provide the necessary support for discussions in activities. Thus, mathematics and reality serve as foundations for mobilizing both mathematical and non-mathematical knowledge through MM, leading to reflection on MM and the applications of mathematics.

From this, it is affirmed that the notions of reality are diverse, as are its dimensions. In most of the analyzed studies, reality is conceived as an everyday fact or even as a topic widely discussed at the moment, a fact belonging to common sense, of interest to the student or teacher. This is the dominant notion of reality in the studies. On the other hand, some studies present a more philosophical and epistemological view of reality, and in this understanding, they provide justifications for its epistemological roots and dimensions, as seen in the studies by Negrelli (2008) and Araújo (2002).

AS NOÇÕES DE 'REALIDADE' NAS PESQUISAS DE MODELAGEM MATEMÁTICA, NA EDUCAÇÃO MATEMÁTICA: UMA REVISÃO SISTEMÁTICA APOIADA PELO SOFTWARE IRAMUTEQ

RESUMO

O objetivo desta pesquisa foi reunir estudos por meio de uma revisão sistemática, com a finalidade de revelar como o fenômeno 'realidade' mostra-se nas pesquisas de MM, na EM, identificar as noções sobre 'realidade', a partir das pesquisas sobre MM, na EM. Para tal, desenvolveu-se uma pesquisa de natureza qualitativa, pautada em uma Revisão Sistemática da literatura pertinente ao campo da Modelagem Matemática, na Educação Matemática. As buscas foram realizadas em sete fontes, dentre elas, bases de dados, portais, bibliotecas virtuais, anais de eventos da área e buscas por citação, a saber: o catálogo de teses e dissertações e o portal de periódicos da Comissão de Aperfeiçoamento de Pessoal de Nível Superior – CAPES, Scielo, Eric Research, o Google Scholar, o Google e a Conferência Nacional de Modelagem Matemática - CNMEM. A Revisão Sistemática foi orientada por um protocolo pautado na estratégia POT (população, Outcome e Tipos de estudos), adaptado para uso em pesquisas qualitativas. O Mendeley foi um recurso utilizado para organizar, gerenciar as referências e excluir as duplicatas dos estudos selecionados para a revisão. Utilizou-se a Análise Textual Discursiva – ATD, apoiada pelo software IRaMuTeQ, para realizar parcialmente a análise do corpus textual. Como resultado desta análise, foram obtidas três categorias: 1) Realidade: seus objetivos e características no âmbito da Modelagem Matemática; 2) Realidade como um dos elementos norteadores da Modelagem Matemática: suas dimensões e seus aspectos epistemológicos; 3) Realidade vista como uma unidade temática interdisciplinar e papel do professor na escolha do tema nas atividades de Modelagem Matemática. Os resultados deste estudo mostraram que as noções de realidade, no contexto da Modelagem Matemática, são plurais. Foram encontradas evidências de algumas raízes filosóficas-epistemológicas da realidade, do platonismo, como, por exemplo, a realidade do mundo sensorial ou realidade platônica e a ideia de realismo. Diante dessas evidências, a partir da imersão na literatura, evidenciou-se as dimensões de realidade presentes nas concepções de Modelagem Matemática, a saber: realidade inicial (realidade dada), realidade intermediária (realidade a ser modelada) e modelo matemático (pseudorealidade).

PALAVRAS-CHAVE: Educação Matemática. Modelagem Matemática. Realidade. Revisão Sistemática. IRaMuTeQ.

NOTES

1 It should be noted that, when convenient, the term Mathematical Modeling will be replaced by MM and Mathematics Education by EM.

2 The term 'reality' was used, which first appeared in the studies of the ICMI (studies on reality and mathematics), according to Bueno (2011).

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Received: Nov. 26th, 2023.
Approved: Nov. 10th, 2024.
DOI: 10.3895/rbect.v18n1.17849
How to cite: ALVES, L. C. S. D.; NUNES, J. M. V. N.; BRAGA, R. M. The Notions of 'Reality' in Mathematical Modeling Research in Mathematics Education: A Systematic Review Supported by IRaMuTeQ Software. **Brazilian Journal of Science Teaching and Technology**, Ponta Grossa, v.18, p. 1-21, 2025. Available at: <https://periodicos.utfpr.edu.br/rbect/article/view/17849>. Access on: XXX.
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