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From Piaget's theory to the construction of mental calculation strategies for addition: an analyisis of the book "Lógica do Cálculo 2"

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ABSTRACT

This article seeks to understand how the authors of the work "Lógica do Cálculo 2" appropriated Piagetian concepts, with emphasis on the operational arguments of identity, compensation, and reversibility, to propose activities that involve mental calculation strategies for addition. The methodological framework drew on Cultural History and History of Mathematics Education, whose central question is to understand how we teach mathematics the way we do. The second volume of the work, intended for 7-8 year-old students and widely circulated in the State of Paraná in the 2000s mainly through training courses offered by the authors to municipal governments, was analyzed. Pre-established categories were used to analyze all the addition activities in the manual, and patterns that repeated themselves were observed. For this article, the three most representative patterns were considered. A brief historical construction was presented on the presence of mental calculation in guiding educational documents in various historical periods, culminating in a closer look at the PCNs (National Curriculum Parameters) in force during the study period. The research concludes that addition activities favor the development of the student's sense of number, as well as the construction of the operational arguments necessary for the development of logical reasoning in 7-8 year-old children, that is, those who are in the transition from pre-operational thinking to concrete logical thinking.

KEYWORDS: Mental Calculation; Piaget; Operational arguments; History of Mathematics Education.

INTRODUCTION

Several circumstances of our daily lives require the use of mental calculation to solve an operation related to some practical situation, such as calculating or checking a change, calculating the percentage of discount on a product or double ingredients of a revenue, among many others (PARRA, 1996). These situations permeate daily life today, but we can find traces of mental calculation in historical sources, indicating that their presence in people's lives and teaching is not recent.

Textbooks, pedagogical manuals used by teachers, curricula and guidelines in each historical period, prove that mental calculation has been part of people's teaching and life for a long time, at times in a more striking way and in others less, with purposes who sought to meet the social demands of each moment.

By studying the history of mathematical education[1], not too distant, we find in the "Parâmetros Curriculares Nacionais" - PCNs (National Curriculum Parameters) for the initial series that "(...) Mental calculation is supported by the fact that there are different ways of calculating and can be Choose the one that best fits a given situation, depending on the numbers and operations involved" (BRASIL, 2001, p. 117). The document points to a relationship between mental calculation and writing:

> Mental calculation procedures constitute the basis of the arithmetic calculation that is used in daily life [...] in mental calculation, reflection focuses on the meaning of intermediate calculations and this facilitates the understanding of the rules of written calculation (BRAZIL, 2001, p. 116-117).

Mental calculation relates to written calculation, so that the understanding of the former, through the different ways of solving an operation, favors the understanding of the second.

Likewise, by focusing on research in the History of mathematics education, we find guidelines for mental calculation teaching at various curriculum proposals at different times.

In Fontes studies (2010), there are official documents from the municipal school system of the city of São Paulo, which reveal the presence of mental calculation as early as 1881. The author reports that this presence does not follow regularity, since from 1882 to 1898, no traces of this teaching were found. From 1899 to 1901, the same author mentions the emphasis on the utilitarian teaching of mental calculation and highlights the fact that it only is present in 1926.

In the 1930s, Fontes (2010) states that with the Francisco Campos reform, the curriculum documents again present the utility mental calculation. Constant use of repetitive exercises and memorization of basic calculations was made, so that "children are used to performing mental memory calculations, valuing speed and practical utility in search of unique solutions, pointing to a conception Traditional teaching" (Ibdem, 2010, p. 70).

According to Fontes (2010), from 1942 to 1961, during the Gustavo Capanema reform, mental calculation remained, having as differential "the expansion of a mere listing of content, for discussions including didactic guidelines" (CONCEIÇÃO, 2021, p. 40).

Almost in the same historical period, Berticelli's (2017) studies analyze mental calculation between 1950 and 1970. They highlight some teaching programs and pedagogical manuals which give utilitarian and practical meaning to the teaching of mathematics that stimulates the use of mental calculation, whose function is to assist the student in problem solving" (avoiding mechanized calculation) and also applying the calculation in everyday practical situations, not only limited to the teaching of the operation" (Ibdem, 2017, p. 62).

For Berticelli (2017), from what he observed in the programs[2] studied, mental calculation is considered "a set of calculation procedures that can be analyzed differently by children in search of exact or approximate results, generally resolved by head" (Ibdem, 2017, p. 65).

From the 1980s and with the movement of the didactics of mathematics, mental calculation was seen as a way of thinking. According to Fontes (2010), between 1985 and 1988, mental calculation presented itself through a gradual work of developing techniques that considered learning as a process of understanding, in search of meaningful learning. In this, the teacher became an articulator, offering situations that allowed the student to build fundamental facts of operations, develop mental relationships and understand operative techniques, including the indication of concrete materials in the development of activities.

With the advent of the PCNs, mental calculation required knowledge necessary for learning, and for sources (2010), it developed after mastery of accounts and arithmetic combinations "such as unfortunate and lists of fundamental facts built with Understanding and not simply with memorization" (FONTES, 2010, p. 129), that is, "activities consisted of building fundamental facts, using estimates and performing calculations from personal strategies" (CONCEIÇÃO, 2021, p. 43).

The activities materialize, to a large extent, in the mathematics textbooks. It is difficult to conceive a math class without supporting the textbook. What interventions are proposed in textbooks regarding mental calculation? In addition to the curriculum proposals of teaching, textbooks become very rich sources to understand how the process of teaching mental calculation took place at a given time, but it is necessary to look beyond the lines, to seek to know what is intended, "(...) It is also necessary to pay attention to what they silence, because if the textbook is a mirror, it can also be a screen" (CHOPPIN, 2004, p. 557). Despite the difficulties encountered, the same author cites that the use of textbooks has been resumed as historical sources, since "the textbook is a material of strong influence on Brazilian teaching practice" (BRASIL, 2001, p. 104).

To Valente (2008),

> [...] The textbook of Mathematics of other times reveals itself as an important means for researching the history of mathematical education. Breaking with the strictly internal analysis of the mathematical contents of these books, the historian of mathematical education will seek to entangle him in a web of meanings, so that he can be seen and analyzed in all the complexity that presents any cultural object. In this web are present multiple elements. Of the conception of the work by the authors, going through the process of how it was produced and suffered the action of editorial houses, reaching the hands of students and teachers and being used by them, the textbook of mathematics may even reveal inheritance of pedagogical practices of the Mathematics teaching, present in our daily school today (VALENTE, 2008, p. 159).

The research cited on mental calculation, from a historical perspective, has a panorama, without contributions that seek to deepen the specific issues of their teaching. As cooperation for the history of mathematical education, this work analyzed mental calculation strategies present in the work "Logic of Calculation" (SIMONS; OLIVEIRA; GOLDSCHIMIDT, 2000) used in 2002, relating them to concepts of theory of Piaget that enable the child to build numerical sense.

Onofre Junior's article "The Teaching of Calculation", published in the Magazine of Pedagogy in 1958, can be considered a first appropriation of Piagetian studies, and announces new times for the pedagogical practices of mathematics in primary education (VALENTE, 2012). Prior to this period, Jean Piaget's works were used in Brazil within the new school (VASCONCELOS, 1996).

In Brazil, at the time of the Modern Mathematics Movement there were several discussions about teaching methodologies and learning process with emphasis on the concepts of Piagetian theory in its structuralist phase. According to Jean Piaget, the structures of the subject's thinking tend to organize themselves by following a mathematical logical model, the mother - algebraic, topological and order structures - that were theorized by the Bourbaki Group (NOVAES, 2012).

For Ubiratam D'Ambrósio (1986), in the 1970s there were profound distortions and a partial and narrow perception of the Piagetian vision that resulted in "Modern Mathematics, which was largely a hurry and distorted application of Jean Piaget's theories to curriculum" (Ibidem, p. 50).

In Brazil, in the 1980s, several research involving a vision of Piagetian development sought to correct distortions, pointed out by Ubiratan d'Ambrósio, on Piagetian theory applied to the teaching and learning studies of mathematics. An example is the research developed by Terezinha Nunes Carraher and collaborators who adapted the concepts to the Brazilian reality. A watershed was the book "Ten in Live, Zero at School" (CARRAHER; CARRAHER; SCHLIEMANN, 2006) in which the researchers applied the clinical method to children of Recife marketers, for example, and concluded that they have all structures cognitive necessary for learning.

In Paraná, there were also examples of appropriations of Piagetian theory by the constructivist movement. For this study, the work "Logic of Calculation 2" (SIMONS; OLIVEIRA; GOLDSCHIMIDT, 2000), was used, recommendedfor the second grade[3] of Elementary School I, published in 2000, authored by Ana Maria Nauiack de Oliveira, Elizabete Goldschimit and Ursula Marianne Simons. The work highlights the construction of logical thinking in children based on their cognitive development and circulated throughout the state of Paraná, both in rural areas[4] and in the capital city[5].

For Valente (2007, p. 39),

> Studying the practices of mathematical education of other times, questioning what was left to them, can mean asking questions for the mathematical textbooks used in past daily lives. They - the textbooks - represent one of the traits the past left us (VALENTE, 2007, p. 39).

From a perspective of the History of Mathematics Education, this article is focusing on the book "Lógica do Cálculo 2" (Logic of calculation 2) (SIMONS; OLIVEIRA; GOLDSCHIMIDT, 2000) as the source and object of the master's research[6], using methodological references from Cultural History, among which

Choppin (2004) and Valente (2007) stand out. It was sought to characterize mental calculation strategies in the analysis based on the Berticelli and Zancan's studies (2021), relating them to Piaget's studies, since as one of the authors of the work states, "(...) He was a researcher, And his interest was not in the child itself, but in an epistemology" and that Piaget offers "important subsidies" so that the child can be understood and assist in his development (SIMONS, 2003, p. 32). Therefore, the central objective of this work is to understand how the authors[7] appropriated[8] Piagetian concepts in the work "Lógica do Cálculo2" (Logic of calculation 2) regarding the mental addition strategies.

Through pre-established categories (bridge by 10, decomposition, compensation, addition and subtraction: inverse operations, basic facts and double memories) all addition activities in the manual were analyzed and patterns that were repeated were identified. Thus, three activities from "Lógica do Cálculo 2" (Logic of Calculation 2) book (SIMONS; OLIVEIRA; GOLDSCHIMIDT, 2000) were selected, highlighting mental calculation strategies and identifying t aspects of Piaget's theory, regarding Identity, Compensation and Reversibility.

Mental Calculation Strategies

Considering that the work presented here was developed within the perspective of the history of Mathematics Education, it was chosen to analyze only the guiding documents that were in force in the period of public work, which were the PCNs (National Curriculum Parameters), which bring, Among the objectives of Mathematics for the 1st Cycle of Elementary School, "(...) Develop procedures of calculation - mental, written, exact, approximate - by observing regularities and properties of operations and the anticipation and verification of results" (BRASIL, 2001, p. 65).

The definitions of Mental calculation receive the most different meanings, divide opinions, provoke doubts and generate expectations (PARRA, 1996). For this analysis, the definition of mental calculation of Zancan (2017) that understands,

> [...] as mental calculation those exact or approximate, which are made mentally, or with notes to support reasoning, which do not depend exclusively on the use of algorithms and counting. These are those who use strategies, numerical logical reasoning, which derive results from other memorized and have their actions validated by numerical and operational properties (ZANCAN, 2017, p. 12-13).

It should be noted that many people associate the speed in resolving algorithms with dexterity in mental calculation, which is not necessarily a reality, because,

> [...] speed is neither a characteristic nor a value, even though it can be a tool in didactic situations in which, for example, allow students to distinguish the calculations that have the results in the memory of those they do not have (PARRA, 1996, p .189).

Speeding is the result of daily understanding and training and mental calculation is more anchored in knowledge, elaboration of strategies than at speed itself. Unfortunately, many relate mental calculation to non -use of pencil and paper, which would prevent the simple record of stages of organized reasoning, seeking to solve the proposed activity. PCNs on the contrary value and stimulate this practice:

> The different procedures and types of calculation relate and complement each other. The written calculation, to be understood, is based on mental calculation and estimates and approximations. In turn, mental calculation strategies, by their own nature, are limited. It is quite difficult, especially in the case of calculations involving multiple numbers, store a large amount of results in memory. Thus, the need for partial results records end up originating written calculation procedures (BRASIL, 2001, p. 115 - 116).

From the moment the student goes on to perform operations mentally, free to manipulate their data and build strategies, without following uniformity, which will later allow their application in everyday situations, will be building a significant learning for each procedure accomplished.

By interpreting Piaget, we understand that when the child is urged to give meaning to the problem to elaborate mental reasoning, it is being induced from the symbol and the real to mental operation, which is directly related to the child's ability to develop and perform Mental calculation, that is, an operation that takes place from concrete to abstract, from the symbolic concrete to the operational mental symbolic. That is why it seeks to give meaning to this knowledge, to elaborate mathematical knowledge from representations and symbols (BERTICELLI, 2017, p. 20).

In the same vein, we can see that the PCNs (National Curriculum Parameters) presented, for the teaching of mathematics, factors that complemented themselves as, the importance of the contents to be worked, the role of the teacher and the disposition of the student to learn, in another words, "[…] It must be considered that not all people have the same interests or skills, nor do they learn in the same way" (BRASIL, 2001, p. 48), and recognize these differences in individual development, conducting teaching so that learning generates new learning.

Piaget says that "[...] the central problem of mathematics teaching is the reciprocal adjustment of spontaneous operative structures proper to intelligence and program or methods related to the mathematical domains taught" (PIAGET, 1983, p. 46), In the same way, PCNs (National Curriculum Parameters) consider the fact that "[...] is not learning that should adjust to teaching, but the teaching that should enhance learning" (BRASIL, 2001, p. 39). Thus, it should be promoted with students activities that gradually evolve, from the easiest to the most difficult, that is,

> One of the first requirements is that students begin to become aware of the procedures they use; They need to know what they know (in order to have this knowledge available) and how they can support themselves in what they know to get other results (...) the calculations that were a tool to solve situations and express what had been made, they become the object of reflection (PARRA, 1996, p. 216).

Activities that value mental calculation represent a way of developing students the ability to operate in different ways, that is, to build varied strategies to solve the same problem situation.

But what is meant by mental calculation strategies? Mental calculation strategies are discussed and studied by several authors, one can mention Berticelli and Zancan (2021), Boler (2018) and Humphreys and Parker (2019) as similar denominations.

Considering the second grade of elementary school I, and the addition operations, one can highlight some categories of mental calculation strategies[9]:

a) Bridge by 10 - According to Berticelli and Zancan (2021), it is a strategy that uses operations that result in number 10, (1 + 9; 2 + 8; 3 + 7; ...) as can be seen by the example In additions: the student learned from the basic facts that 8 + 2 = 10, now when adding 8 + 6, he can say (8 + 2) + 4 = 10 + 4 = 14;

b) Decomposition - When you make 6 + 8, the student can break down 8 in 4 + 4, so will have (6 + 4) + 4 = 10 + 4 = 14, ie knowing the basic facts the student decomposes one of the numbers And using the bridge by 10, or even the basic facts, makes the operation easier to perform. Humphreys and Parker (2019) also bring decomposition in subtraction, when proposing to decompose the subtracting, for example, 63-28, decomposes 28 in 20 + 8, which makes 63-20 = 43, now, can be Decomposing 8 in 3 + 5, so the student will do 43 -3 = 40 and 40 -5 = 35. For the authors "decompose the subtracting uses the ease of students with subtraction with multiples of 10 and their fluency with small numbers" (HUMPHREYS; PARKER, 2019, p. 49);

c) Compensation - presents the possibility of, when making 9 + 5, the student remove from one portion and put in another, ie 9 + (1 + 4) = (9 + 1) + 4 = 10 + 4 = 14 (ZANCAN, 2017). Humphreys and Parker (2019) call this "take and give" strategy. For Boaler (2018), this ability to interact with numbers flexibly and conceptual is characterized as numerical sense. According to the author, students with a numerical sense developed can solve operations simply and easier, using the flexibility of numbers;

d) Addition and Subtraction: Inverse Operations - "It consists in recovering random memory results and using the reverse property of operations" (ZANCAN, 2017, p. 22), such as to solve 7 - 3 = 4, the student can Seek in your memory of basic facts, the information that 3 + 4 = 7, and thus resolve subtraction through an addition, because "the idea of never having to subtract delights many students" (HUMPHERES; PARKER, 2019, p. 50).

To build these strategies, Berticelli and Zancan (2021) listed four categories of knowledge that are essential:

a) Basic facts - These are operations where the results do not exceed dozens, ie those where the operation is performed only in the units. It can be cited as examples 2 + 3 = 5, 3 + 4 = 7, 12 + 7 = 19, proposed also defended by the PCNs "Students build the basic facts of operations (calculations with two terms, both less than ten), constituting a repertoire that supports mental and written calculation" (BRASIL, 2001, p. 68);

b) Network of Relationships 10 - Operations involving 10 in installments or results, and their multiples, as examples we have 1 + 9 = 10, 2 + 8 = 10, 11 + 10 = 21, 10 + 15 = 25;

c) Double memory - Usually the memorization of double numbers less than 20, examples 8 + 8 = 16, 12 + 12 = 24, 18 + 18 = 36. This memory type is used for operations of the type: 8 + 9, where the thought is, if 8 + 8 = 16, then, 8 + 9 = 17, because 8 + 9 = 8 + 8 + 1 = 17.

d) Decomposition - memorized operations that allow to recognize all possible decompositions of a number less than 10. Some examples are: 3 = 1 + 2; 4 = 1 + 3 = 2 + 2; 5 = 1 + 4 = 2 + 3. This type of decomposition is relevant as the child needs to know which one should use in a given situation. For example, to make 8 + 7, using the bridge by 10, it is necessary to know that 7 = 1 + 6; 7 = 2 + 5; E, 7 = 3 + 4 And in this case use 7 = 2 + 5, because 8 + 2 = 10. then, 8 + 7 = 8 + 2 + 5 = 10 + 5 = 15. If the memory was used Doubles could be done in two ways: 8 + 7 = 7 + 7 + 1, and conceive that 8 = 7 + 1, or even 8 + 7 = 8 + 8 - 1, considering that 7 = 8 - 1. From these examples, the relevance of working on decomposition is justified, as this knowledge is the basis for the construction of strategies.

It should also be noted that the activities that stimulate the basic facts offer the student the possibility of creating the memories, which will be used at different times, with different strategies, besides being "(...) that the student gets confidence in His own ability to learn mathematics and explore a good repertoire of problems that allow him to advance in the process of concept formation" (BRAZIL, 2001, p. 70).

PIAGET AND LOGICAL REASONING

Jean Piaget, an important representative of learning psychology, focused his investigations on cognitive structures, his theory being called Genetic Epistemology. For Piaget (1983):

> Logical-mathematical reasoning is necessary in various domains of knowledge [...] and most importantly, this knowledge is demonstrated and not only transmitted or delivered to the child, states that it should be built on each and interactively and interactively (PIAGET, 1983, p. 12).

For Ursula Marianne Simons[10] logical thinking starts from the premise that "it is not content in itself, but it gives conditions for the thinking and content of the various sciences to be consistent, consistent" (SIMONS, 2003, p. 31).

According to Piaget (1983), the environment and situations used for learning to occur, are of fundamental importance for the development of mathematical logical reasoning, because arithmetic should be reinvented by the child,

> The social environment and the situation that the teacher creates are crucial in the development of logical-mathematical knowledge. Since this knowledge is built by the child through reflective abstraction, it is important that the social environment encourages the child to use it. According to Piaget, all children of normal intelligence can learn arithmetic. Arithmetic is something children can invent, not something that can be transmitted. [...]. If mathematics is so difficult for many children, it is because it is imposed on them, without any consideration for the way they learn or think (KAMII; DECLARK, 1988, p. 63).

The learning environment is highlighted by Nacarato, Mengali and Passos (2021) as the one that allows the dialogical relationship between students and teachers in the classroom. This is where the students' thinking is allowed to hear. In it, the student is able to communicate intellectually and produce mathematics.

For the authors “in this environment, (...) the processes of thought and the strategies of students need to be valued; The absolutism of 'right and wrong' can give way to discussion, dialogue” (Ibdem, p. 38-39).

Piaget (1983) studies also define some important concepts for child development, which are the processes of assimilation, accommodation and balance. Assimilation-It is understood by the fact that, upon receiving a new idea, the subject adds this to the knowledge he already has. Accommodation - are the transformations that internal systems need to make to add new knowledge. Being both processes necessary for cognitive development, it is observed that the new knowledge and transformations caused in internal systems cause a certain cognitive imbalance that soon becomes balance in the child he learns, this Piaget process calls-balancing (FONSECA, 2014).

Piaget also suggested the age groups for each stage of the individual's development that can be organized as follows, according to Ramozzi-Chiarrotino (2005):

> Sensory-motor period (from birth to 1 year and a half/2 years, on average), in the course of which the systems of schemes that prefer future operations, but without any operative reversibility are constituted; Period of intuitive thinking (from 2 to 7 years, on average), in the end sensorimotor actions begin to imply representation, mental image, noting here the presence of semi-reversible regulations; The period of “concrete operations” (7 to 12 years old, always on average), in the course of which a certain reversibility is achieved in the formation of the first operative structures and that involves an implicative aspect; And in a fourth period, that of propositional operations, completely reversibility is reached and the distinction between timeless and temporal phenomena, between mechanical and historical phenomena, that is, reversible and irreversible phenomena (RAMOZZI-CHIAROTTINO, 2005, p. 19).

This work will stick to the period of concrete operations, which is why it was chosen to analyze the book “Lógica do Cálculo 2” (Logic of Calculation 2) (SIMONS; OLIVEIRA; GOLDSCHIMIDT, 2000), which was intended for the 2nd grade of elementary school I, which covers children From 7 to 8 years, because, according to Piaget:

> The age of 7-8 years on average marks a decisive moment in the construction of the instruments of knowledge. The internalized or conceptualized actions with which the subject should have so far content to acquire the category of operations, while reversible transformations modify certain variables and retain others by invariants (PIAGET, 1983, p. 30).

For Piaget and Inhelder (1975), the structures of classification, serials and term correspondence are structures that, if well developed, stimulate the transition from preoperative thinking to concrete logical thinking.

Simons (2003) notes that, from the sensorimotor period, the child has a long way to go to build his logical reasoning. With the proper stimulus, it is expected that around six or seven years it has already had a structured logical reasoning so that it can develop flexible and creative learning. The author also highlights the fact that they observe, quite often, students aged eight to ten years when this organization did not occur or was not completed. This results in difficulties in conserving physical quantities, classification or inclusion of classes, a fact due to the emphasis that most schools gives greater importance to a curriculum of content to be overcome over the construction of the child's logical thinking. Often, schools boast that students already know how to read, however, the same, most of the time, cannot classify or serial, which brings serious learning difficulties throughout school life.

Education is not in itself, it needs intentional interventions of teachers. According to Simons (2003, p. 16-17) “the higher the quality of these interventions, the greater the autonomy and creativity of the individual, preparing him for the active construction of the world he wants”.

Piaget and Szeminska (1975) state that in the study of the additive composition of a numerical order, they are successively employed three arguments[11] parallels: identity, compensation and reversibility.

Simons (2003) presents that between two and six years of life, the child is already moving, has mastery of language, but has no mastery of formal logic, does not realize the reasoning of identity, which makes it understand that things do not change when They change their position, this thought called by Piaget.

Piaget and Inhelder (1975) have two types of identity argument, one in a positive form that simply states that it is the same thing, and another, from the negative point of view, where it can be said that nothing has been taken or added. Identity, from this perspective, can be understood as follows (7 + 1) = (1 + 7).

About compensation, Piaget and Szeminska (1975), write that:

> On the other hand, the passage of classes to the numbers is produced as soon as A1, A2, A'1, A'2 are considered no longer as simple collections to present each in their qualitative individuality, but as units that can be equal to without being identified (equalization of differences) or reduced in their inequalities to a unit system that serves as a common measure. Indeed, so, thanks to this equalization of differences, each grain or set of grain becomes a unit both equal to the units of the same category and distinct by its order of enumeration, so operations acquire a numerical meaning. If we call the difference between A1 and A2 or between A'2 and A'1, ie d = (a1 - a2) = (a'2 - a'1), then the subject establishes that: a1 = a ´1 = (a2 + d) = (A'2 - D), ie 4 = 4 = (1 + 3) = (7 - 3) (PIAGET; SZEMINSKA, 1975, p. 260).

From Piaget and Szeminska (1975) we understand that compensation is the idea that,

$$4 = 4$$

$$(1 + 3\ ) = (7 - 3)$$

In this sense we can think of the different compositions of a number, for example 9,

$$9$$

$$(1 + 8\ ) = (10 - 1\ )$$

$$(\ 2 + 7) = (11 - 2)$$

$$(\ 3 + 6\ ) = (12 - 3)$$

$$(\ 4 + 5\ ) = (\ 13 - 4)$$

And so on.

From the Piagetian perspective, reversibility seems to mark the completion of the process of building operative structures, ensuring a character of logical need, "(...) the reversibility that characterizes operative structures marks the finishing of these approximate compensations, manifested by regulations" (PIAGET; INHELDER, 1975, p. 119). For this study, reversibility is the idea that (3 + 4) = 7, then (7 - 4) = 3 or (7 - 3) = 4.

For Kamii and Declark (1988), between seven and eight years old, children's thinking becomes flexible enough to be reversible, so reversibility is the ability to mentally perform opposite actions simultaneously.

Briefly, it can be understood that:

> [...] The additive hierarchy of the classes, the serialization of relationships and the operative generalization of number (that means, the construction of numbers that exceed intuitive integers, 1, 2 to 4 or 5) constitute approximately synchronously , around 6 to 7 years, when the child's reasoning begins to exceed the initial pre -logical level: is that the class, asymmetrical relationship and the number are, the three, complementary manifestation of the same applied operative construction , be it the equivalences and differences gathered. Indeed, it is at the moment that the child has been able to make the intuitive assessments of the beginning, thus reaches the level of reversible operation, which it becomes simultaneously capable of including, serial and enumerating (PIAGET; SZEMINSKA, 1975, p. 253).

Thus, it is observed that the additive mechanism that matters here is organized in three phases. In the first of these phases, the child does not understand the necessary compensation of additions, that is, adding a certain number of elements to the Mount A, they do not expect Mount A to decrease by the same amount. In the second phase, the child becomes aware of this balance, but only in the intuitive plane, that is, outside the figures has no other means to verify equalities, nor, therefore to predict the result of additions. Finally, during the third phase, there is an operative management of transfers and, consequently, well-regulated reversibility (PIAGET; SZEMINSKA, 1975).

THE WORK “LÓGICA DO CÁLCULO 2” (LOGIC OF CALCULATION 2)

The work “Lógica do Cálculo 2” (Logic of calculation 2) (SIMONS; OLIVEIRA; GOLDSCHIMIDT, 2000), was published in 2000 and acquired in 2002, by the Municipal Department of Education of Maripá, a municipality of western Paraná, Brazil, after the participation of teachers in continuing education course with one of the authors, Ursula Mariane Simons. Upon noticing the great potential of the material, the teachers presented implementation proposals in the municipality, which was approved and the collection became part of the pedagogical material of municipal schools, until 2007. Even after 15 years of this project in the city of Maripá, teachers from the municipality use the material in their classes as a complement to the textbook added in schools.

The work is authored by Ana Maria Nauiack de Oliveira - Degree in Mathematics from the Federal University of Paraná, Master in Education from the Federal University of Paraná, with specialization in Psycho-pedagogy, Professor of Basic Education and Higher Education, works in Psycho-pedagogical Clinic, in the area development of mathematical logical reasoning; Elizabete Goldschimit – a visual artist with a degree in Fine Arts at the Federal University of Paraná, a teacher of Basic Education and Art Schools for Children and Adolescents, conducted research in the area of education in various European countries; and, Ursula Marianne Simons - Psychologist from the Federal University of Paraná, a specialist in Psycho-pedagogy, Ludotherapy and Psychomotricity, a teacher of higher education, works in a psycho-pedagogical clinic and conducted research in several European countries, regarding the teaching methodology in the early grades of basic education.

The work has a total of 224 pages and the necessary knowledge for the elaboration of mental calculation strategies, and the four operations permeate the material in a way and coming. Operations are not linearly, but if it perceives a continuous movement, in which the addition is preceded from the subtraction, then the addition is resumed, the contents are presented, in another words, the contents communicate, and a It is a prerequisite for the other more advanced.

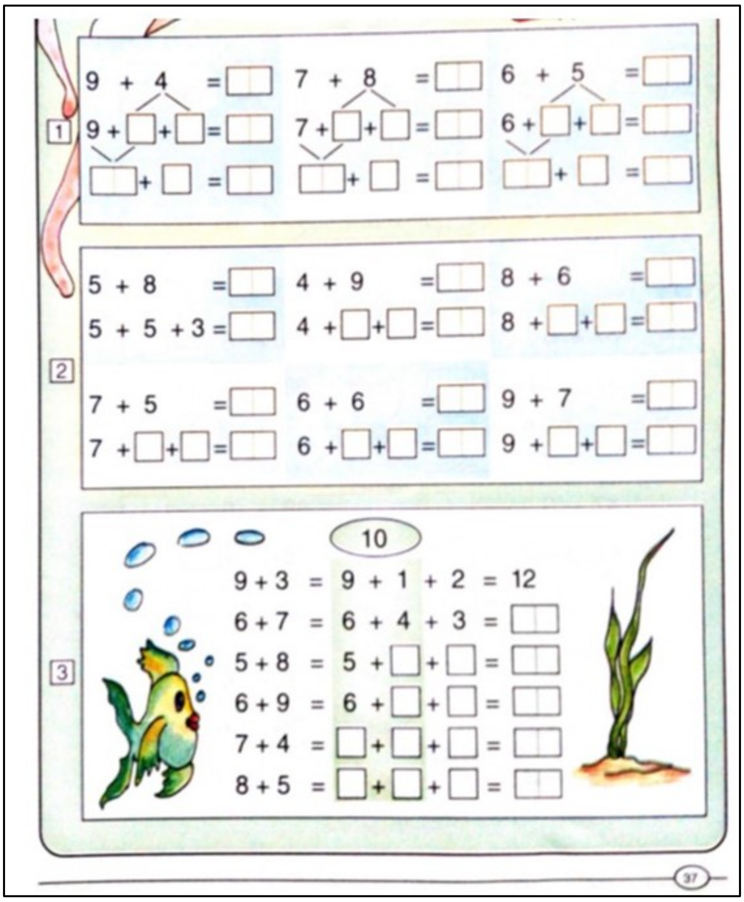
For this text, the objective was to conduct an initial study, highlighting activities that present mental calculation strategies, as well as the concepts listed by Piaget. Only three figures were shown, as these concepts eventually repeat themselves throughout the work. The relevant fact is the presence of strategies and the appropriation of the theoretical aspects of Jean Piaget's theory by the authors.

Following are the selected activities of the work, followed by an analysis of mental calculation strategies of the addition and relationships with Piaget's studies.

In all the activities proposed in Figure 1, the bridge by 10 stands out, considered an addition strategy (BERTICELLI; ZANCAN, 2021). For this strategy it is necessary to have knowledge of decomposition, as one of the numbers will be decomposed for the other to complete the dozen. For example: (7 + 5) = (7 + 3 + 2) = (10 + 2) = 12. It is necessary to know that: (7 + 3) completes the dozen, which (5 = 1 + 4), (5 = 2 + 3), (5 = 4 + 1) and (5 = 3 + 2); And in this case, choose (5 = 3 + 2) to be able to use the three and add to seven.

Regarding Piaget's studies, one can observe the reasoning of identity and compensation. The identity is proven because the student realizes that (2 + 3) = (3 + 2). Compensation is characterized as the student observes that (7 + 5) = (10 + 2).

Figure 1: Ponte Activity by 10



Source: Simons, Oliveira and Goldschimidt (2000, p. 37).

In Figure 2, we observed the training of basic facts and training of operations up to 100, where it is already possible to trigger the bridge strategy by the 10, which has already been demonstrated. It is necessary for the student to use the identity and compensation reasoning.

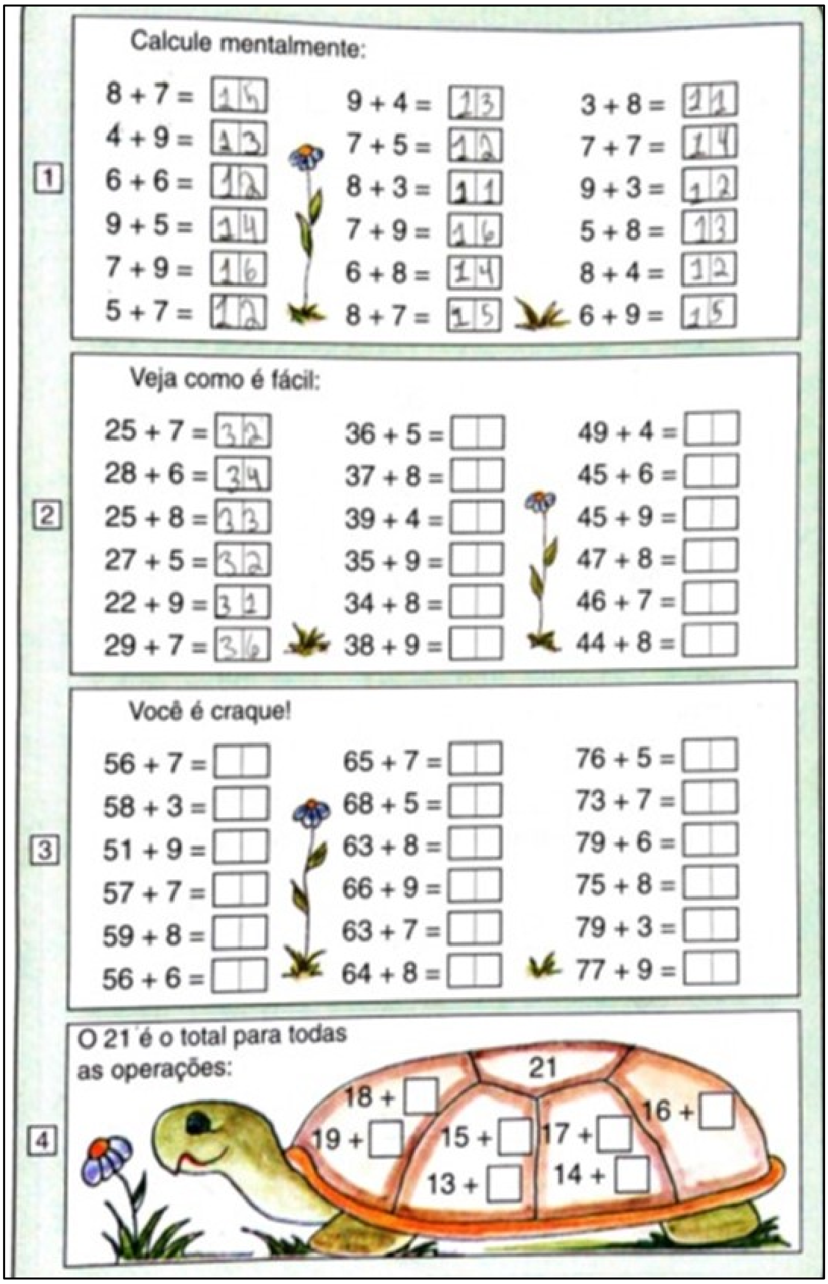
Analyzing only activity 1, it is observed:

- Compensation when the authors offer: (8 + 7) = (6 + 9); (6 + 6) = (5 + 7), (7 + 5) = (9 + 3); (9 + 5) = (6 + 8) = (7 + 7);
- Identity is found in operations: (4 + 9) = (9 + 4); (5 + 7) = (7 + 5); (8 + 3) = (3 + 8).

In activities 2 and 3, we also observe the use of the bridge by 10, but with a magnitude of the numbers.

In activity 4, the authors offer number 21 and some incomplete mathematical expressions in which students should complete them. We recognize here the concept of compensation, since all expressions will result in 21. For example: 21 = (19 + 2) = (18 + 3) = (17 + 4) = (16 + 5) = (15 + 6) = (14 + 7) = (13 + 8).

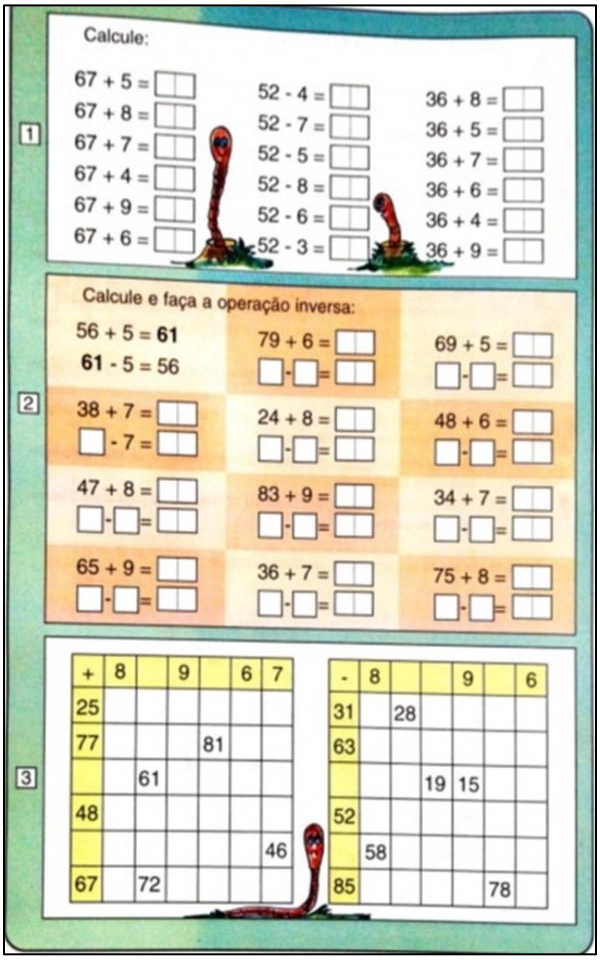
Figure 2: Basic facts.



Source: Simons, Oliveira and Goldschimidt (2000, p. 40).

Observing Figure 3, activity 1, we find simple addition training in columns 1 and 3. It is observed that one of the plots is fixed, allowing one result to be used to compose the other. For example, if (67 + 5 = 72) then (67 + 6 = 73). It is possible to trigger the bridge strategy by the 10, which has already been presented: (67 + 6) = (67 + 3 + 3). On this page are the principle of reversibility (PIAGET; SZEMINSKA, 1975), in which the student can distinguish subtraction as the inverse of addition. This principle is present in activity 2, when the authors suggest that the reverse operation is done indicating that: (56 + 5) = 61 then (61 - 5) = 56.

Figure 3: Inverse operations



Source: Simons, Oliveira e Goldshchimidt (2000, p. 148).

FINAL CONSIDERATIONS

Understanding how mathematics was taught in different periods of history by examining school library records or methods of private teachers can help us understand how teaching and learning were characterized in the past and how this knowledge can inform decision-making today.

By examining cultural history, mental calculation strategies, and Piaget's studies, we can see that mental calculation was once an integral part of classroom activities during a certain period in the history of mathematics education. The material analyzed demonstrates that these activities were informed by Piaget's theories of identity, compensation, and reversibility, which are essential for understanding arithmetic and building numerical sense.

It can be concluded that the quality of interventions on mental calculation in the book "Lógica do Cálculo 2" (Logic of Calculation 2) enhances understanding of the decimal numbering system and helps children develop a meaningful understanding of numbers. This suggests that the authors drew from Jean Piaget's concepts in their work.

However, there are still questions to be answered. Do all the books in the collection incorporate the same knowledge and strategies? Are there other authors besides Piaget who influenced the work? What other knowledge and strategies are proposed in this work?

To answer these questions and to "do exactly what every successful researcher has always done: move forward" (BOOTH; COLOMB; WLILLIAMS, 2008, p. 31), much more research is needed. A fascinating journey through the history of mathematical education awaits us.

DA TEORIA DE PIAGET À CONSTRUÇÃO DE ESTRATÉGIAS DE CÁLCULO MENTAL PARA ADIÇÃO NA OBRA LÓGICA DO CÁLCULO

RESUMO

O artigo que se apresenta, busca compreender como as autoras da obra Lógica do Cálculo 2, se apropriaram de conceitos piagetianos, com destaque aos argumentos operatórios da identidade, compensação e reversibilidade, para proporem atividades que envolvam estratégias de cálculo mental da adição. O referencial-metodológico buscou aportes da História Cultural e História da educação matemática cuja questão central é compreender como ensinamos matemática da forma que ensinamos. Analisou-se o segundo volume da obra destinado aos alunos de 7-8 anos e que circulou nos anos 2000 no Estado do Paraná principalmente por meio de cursos de formação ofertados pelas autoras as prefeituras municipais. Por meio de categorias pré-estabelecidas analisou-se todas as atividades de adição do manual e percebeu-se que havia padrões que se repetiam. Para esse artigo considerou-se as três mais representativas. Apresentou-se brevemente uma construção histórica sobre a presença do cálculo mental em documentos norteadores da educação em vários períodos históricos culminando com um olhar mais atento aos Parâmetros Curriculares Nacionais (PCN) em vigor no período de estudo. A pesquisa conclui que as atividades de adição favorecem o desenvolvimento do sentido do número por parte do aluno, bem como a construção dos argumentos operatórios necessários para o desenvolvimento do raciocínio lógico em crianças de 7-8 anos, ou seja, aquelas que se encontram, segundo Piaget, na transição do pensamento pré-operatório para o pensamento lógico concreto.

PALAVRAS-CHAVE: Cálculo mental; Piaget; Argumentos operatórios; História da educação matemática.

NOTES

1 For Valente (2013, p. 24) “In this text we distinguished “mathematics education” from “mathematical education”. The first expression designates the recent academic field, a place of investigations on mathematics teaching and learning. A founding reference, in Brazil, this field can be given by the creation of SBEM- Brazilian Society of Mathematics Education, in 1988. The second expression refers to the teaching and learning processes of mathematics since immemorial times, thus constituting itself in research theme of studies related to the history of mathematics education. In any case, the distinction is necessary so that it is not thought that by 'History of Mathematics Education' were only allocated to post- Anan studies 1980, or even restricted to the history of the research field. ”

2 The author analyzed state programs: Mato Grosso, Minas Gerais, Paraná, Rio de Janeiro, Rio Grande do Sul, Sao Paulo and the Federal District.

3 The amendment to the term years, occurs by Law No. 11,274 (BRASIL, 2006), from February 6, 2006, expanding elementary school to nine years and established a period of implementation by systems until 2010.

4 There were records of the purchase of material by the City of Maripá from 2002 to 2007.

5 Certificates of continuing education courses, presented by teachers from the municipal network of Curitiba at the time.

6 This text points to partial results of the Master Research ongoing (Post - Graduation in Science Education, Mathematics Education and Educational Technologies - PPGEMETE - UFPR - Palotine Sector).

7 Author Ursula Marianne Simons gave an interview about the work in December 2021, which allowed us a better understanding of the production of the book “Lógica do Cálculo 2” (SIMONS; OLIVEIRA; GOLDSCHIMIDT, 2000). This interview will be part of the developing dissertation.

8 For Chartier (2002, p. 68), appropriation refers to “a social history of uses and interpretations, related to their fundamental determinations and inscribed in the specific practices that produce them”.

8 There are several mental calculation strategies for addition. In this section we present those we find in the activities of the “Lógica do Cálculo 2” book (SIMONS; OLIVEIRA; GOLDSCHIMIDT, 2000).

9 One of the authors of the analyzed work.

10 For this reason we chose to analyze the “Lógica do Cálculo 2” book (SIMONS; OLIVEIRA; GOLDSCHIMIDT, 2000), which was intended for the 2nd grade of elementary school I, which covers children from 7 to 8 years.

11 These same arguments are cited by Ursula Marianne Simons, one of the authors of the “Lógica do Cálculo 2” book (SIMONS; OLIVEIRA; GOLDSCHIMIDT, 2000), in the interview that granted us in December 2021.

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