Revista Brasileira de Ensino de Ciência e Tecnologia
ISSN: 1982-873X

https://periodicos.utfpr.edu.br/rbect

# Visualizing geometric shapes in bushes

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## ABSTRACT

In this article, qualitative research is presented, which aimed to analyze how two distinct groups visualized constant geometric shapes in figural records of bushes found in the urban environment of a city in the metropolitan region of the capital of Rio Grande do Sul. This analysis includes two of the six images investigated, which are close to a paraboloid and a torus. The first group had 29 participants in a workshop given by the author at an international event held in Colombia, with students and/or professors from different nationalities and levels of education. The second group, with 14 participants, all Brazilians and members of a study and research group led by the author. The results showed limitations of the participants in visualizing geometric shapes, especially spatial ones. Most of them identified more flat geometric objects than the two spatial geometric shapes, and few identified the paraboloid and the torus.

**KEYWORDS**: Paraboloid. Toru. Visual perception. Urban environment.

## INTRODUCTION

This article consists of an expanded version of the work presented at the VIII SIPEM- Seminário Internacional de Pesquisa em Educação Matemática (International Seminar on Research in Mathematics Education). The results of the author's research on the formation of geometric thinking are analyzed, particularly involving skills such as creativity and imagination, as well as visual perception.

Since Euclides, geometry awakens to the needs of man in his day-to-day life, such as land measurements, advancing to other fields, such as analytical geometry, with Descartes, and even the creation of non-Euclidean geometries. The latter came to contribute to the good life of individuals, that is, in air navigation, for example, there is the elliptical, in which the straight lines no longer have the same meaning as established by Euclides. In this geometry, a line is visualized in its figural representation by circles on the surface of a sphere.

The rupture in exclusively Euclidean thinking led to the visualization of triangles whose sum of the measures of the internal angles is greater than 180° (elliptical or spherical geometry) or smaller than that (hyperbolic geometry). Also, in the first one, the existence of triangles with three right angles and the notion of biangle can be seen. Those who imagine the existence of only these geometries are wrong, as many others have emerged over time: topological, fractal, taxi, finite, etc.

Piaget and Inhelder (1993), with regard to the intuition of shapes in geometry, indicate that, around 3.6 to 4 years of age, the beginning of abstraction of shapes occurs. For them, “[...] curiously, the first forms recognized are not Euclidean, but topological [...]” (p. 36). By making circular or square objects available to children, their tactile exploration becomes important, for example, by contouring the object, which allows them to identify differences between both. Only after initiating the abstraction of forms, ceasing to touch the object, is that the individual manages to start with Euclidean relations. The authors state the importance of “[...] knowing what the abstraction of forms consists of and why topological forms are psychologically simpler than Euclidean forms” (p. 37).

Similarly, the concept of distance could be brought to the real or concrete world, that is, how displacement is carried out in urbanized cities in well-defined blocks ('squares'). In this sense, is not present the conception of students (at various levels of education, including higher education or even among mathematics teachers) that the shortest path between two points is represented by a straight line segment ('a diagonal of this square'). In Taxi Geometry, for example, the distance between two opposite points in a block is given by the sum of two distances (that of the two legs that, together with the diagonal, form a right triangle). It is understood that this is a situation much more consistent with today than the old measurement of land with the invasion of the Nile, as stated in history books.

Perceiving geometry in the world, that is, how formal geometry can be visualized in constructions, decorations, nature, is a good way to analyze whether its teaching is not limited to the use of formulas and the reproduction of theorems and definitions. Based on these preliminary considerations, this article is justified, which aims to associate geometric shapes with ornamental shrubs observed by the author of the article in an urban environment (LEIVAS, 2021).

## THEORETICAL ASSUMPTIONS

A study on creativity in mathematics education is presented by Meissner (2015), which indicates mental processes in the quest to promote creative thinking, that is, alludes to the need to involve, in the classroom, challenging problems and their respective natures. For the author, the proposed and developed activities must integrate, in a natural way, spontaneous ideas of individuals, which may be unconscious and intuitive with regard to knowledge (known or to be known).

With regard to teaching mathematics, Kandemir and Gür (2007) indicate that, in Turkey, similarly to what happens in Brazil, high school teachers prefer to teach problem solving through algorithmic approaches rather than using new teaching methods. They state that they believe that “practicing examples in this way is the best preparation for entering university” (p. 111). Before conducting a course with a group of teachers in a research project about students' views on creativity training, the authors presented twenty-two different creativity techniques. Some of them are highlighted here: a) Brainstorming; b) Creative pause; c) Creative exchange; d) Imagination.

It was deemed important to highlight these techniques for the following reasons.

- Brainstorming by stimulating the debate between the components of a group regarding a given theme/problem, as in the first stage of problem solving derived from Polya (2006). In the sequence, also for the attempt to resort to creativity to solve problems.
- Creative pause for being directly linked to psychology and for having already been highlighted by Poincaré with regard to mathematical creation. Thus, it enables the teacher and the group to have a collaborative feeling.
- Creative exchange for addressing the exchange of creative ideas between individuals or groups.
- Imagination for considering planning to solve a problem.

Such aspects are highlighted here because they meet the research interests of the author of this article in terms of imagination, intuition, visualization and creativity.

When dealing with science and logic under the eyes of some mathematical educators at different times, Poincaré (1969) rescues conceptions about intuitionism. For him, it is not minds that have changed, but intuitive ideas. The thinker asks: “What is the cause of this evolution?”. For him, “Intuition cannot give us rigor, not even certainty, this has been increasingly recognized” (p. 207, free translation). The author cites many situations that have occurred throughout history, such as the case that every continuous function has a derivative and that, later, it was realized that it does not occur as in cusp points, as can be seen in Leivas (2021).

In his Fundamentals of Geometry, Hilbert (2003) reiterates the following idea:



> Mathematics evolves, in a sense, towards conversion into an arbitral tribunal, a court of last resort that will decide questions of principle – and on concrete

> bases such that universal agreement is attainable and all assertions can be verified.
>
> Also the assertions of the recent doctrine called <<intuitionism>>, however modest, can, in my opinion, obtain their certificate of justification only through this court. (HILBERT, 2003, p. 254).

It is understood that such a court consists of making available to the scientific community the 'findings' in a research, such as those that are being carried out to intuitively and creatively visualize geometric shapes found in urban environments and certify them through Dynamic Geometry in order to confront them with formal concepts (LEIVAS, 2014; LEIVAS, 2021).

For Fischbein (1987), intuition can be defined as

> an idea that possesses the two fundamental properties of a concrete, objectively given reality; immediacy, that is, intrinsic evidence and certainty, not conventional formal certainty, but practically meaningful, immanent certainty (p. 21).

According to Davis and Hersh (1995), intuition has the following meanings: 1) it is the opposite of rigorous, although it is not, as such, very accurate; 2) it is visual; 3) it is plausible or convincing in the absence of proof; 4) it means incomplete, as it is complemented by the logic of demonstration; 5) it is based on a physical model or on important examples, that is, almost heuristic; 6) it means unified or integrated as opposed to detailed or analytical.

Contributing with the visual sense of intuition, the author of this article considers the ability of visualization to be important for the development of geometric thinking and believes that it is not innate, but can be developed in the course of the entire formation of the individual. For Leivas (2009, p. 111) “visualization is a process of forming mental images, with the purpose of constructing and communicating a certain mathematical concept, with a view to assisting in the resolution of analytical or geometric problems”. For the construction of this concept, the author associates it with imagination and creativity.

It is considered opportune to establish connections between the aforementioned skills, especially with regard to the representation of space, that is, to seek connections of geometric intuition (what the individual perceives in the environment around him) and where the imagination leads him in the scope formal, in this case, to geometric shapes. According to Piaget and Inhelder (1993, p. 467), “Now, it is true that spatial intuition partly resorts to the senses and imagination [...]” (emphasis added). For example, what is a point or a line in plane or spherical geometry or even that of a taxi, if not appealing to intuition and imagination. It is necessary that creativity be evoked in the teacher to address these geometric themes, so that the individual can imagine, for example, a circumference represented by a square when the metric used is that of the taxi (a square ball), (LEIVAS, 2014).

Hadamard (2009, p.114) poses the following question: “Can imagery[1] be educated?” when discussing the work of Ticheber. In addition to verbal language, it indicates the importance of visual images in thought, obviously allied to auditory and musical images. Also, according to the author, visual imagery “is always at my disposal and I can shape and direct it at will”. Then, complements Hadamard

(2009), “When I read any book, I instinctively arrange the facts or arguments according to a visual model and I have as many possibilities to think in terms of this model as I have to think with words” (p. 114).

Therefore, exploring the visual imagination seems to be a good indication for appreciating the geometry involved around the individual, since the mental processes are touched upon (intuition, creativity) for the formalization, through geometric language, of geometric concepts such as those of some surfaces, combined with the resources of Dynamic Geometry software.

## METHODOLOGICAL PROCEDURES

This article is based on the work presented during the VIII SIPEM. According to Moreira (2011), the research methodology in science has as one of its paradigms being “[...] derived from the humanistic area with an emphasis on holistic and qualitative information and interpretive approaches” (p.74). Because it is about analyzing how individuals interpret images, photographed by the researcher, of some bushes located in an urban environment, connecting them to spatial geometric shapes, it is understood to be an interpretative qualitative approach. Furthermore, the author states that “the interest of this research is in an interpretation of the meanings attributed by the subjects to their actions in a reality” (Idem, p. 76).

Based on these considerations about qualitative research, the researcher presented images of shrubs that decorate sidewalks and flowerbeds in an urbanized city. In his eyes, such images were configured as representations of geometric shapes such as paraboloid, ellipsoid, cone, torus, etc.

With regard to data collection, these, “[...] must be collected and recorded with the necessary rigor and following all field research procedures. They must be worked on, through rigorous analysis, and presented in qualified reports” (SEVERINO, 2016, p. 128).

In this direction, for a rigorous approximation of the visualized image with the geometric one, GeoGebra was used, so that, when inserting the image in the software, it was possible to find such a connection. The effect of this use may allow future teachers to develop visual skills together with their students, that is, to think geometrically.

When offering a short course at an international event in February 2021, the researcher presented a Google Forms with images photographed by him, associating them with geometric shapes and the participants should justify their choices and, thus, make it possible to analyze the answers, which were framed in two categories: flat figures and spatial figures. In all, twenty-nine participants answered the questionnaire, which will be called $PC_1$, $PC_2$,..., $PC_{29}$. Among the participants, there were undergraduate, specialization, master's, and doctoral students and professors from eighteen institutions of different nationalities.

In a second moment, the same questionnaire was applied to the study group led by the author of the article. Fourteen individuals participated, from undergraduate students to doctors from different institutions and Brazilian states. Participants in this second group were designated by $PB_1$, $PB_2$, ..., $PB_{14}$.

We highlight the fact that, for the analysis of the respondent, a code name was requested, since some personal data was requested, such as education, institution, etc.

For the questionnaire, six images were randomly collected by the researcher during his morning walks, during the COVID-19 pandemic period, in the metropolitan region of the capital of RS. Two of them were chosen for this article. It should be noted that: "[...] images, with or without sound accompaniment, offer a restricted but powerful record of temporal actions and real events – concrete, material" (LOIZOS, 2015, p. 137). The author also reinforces the fact that social research "[...] can use, as primary data, visual information [...]". As there was interest in visualizing specific geometric shapes, sound images were discarded. Therefore, the skills discussed in the theoretical foundation are well placed, since the objective of the research was to analyze how individuals visualize geometric shapes found in bushes in an urban environment and associate them with spatial geometric shapes.

Each question had a figure and, below it, the statement. The participant had to choose and indicate whether he saw any geometric shape on the pictures of bushes presented (yes or no), as well as justifying and naming the perceived geometric figure. The fact that the mini-course was applied in a Spanish-speaking country stands out and, therefore, the utterance was presented in the language of the country.

"Abajo de la imagen, indica si percibes alguna forma geométrica. En caso afirmativo, identifica el nombre de la forma. Si posible, justifica brevemente tu respuesta". (Below the image, indicate if you perceive some geometric shape. If so, identify the name of the form. If possible, briefly justify your answer).

Figure 1 – photos of two of the images

Source: Researcher's archive (2021).

The choice for the two figures does not have a specific criterion. The same form was presented to the second group, changing only to the Portuguese language. Next, the analysis of the responses obtained is carried out.

## DATA ANALYSIS

Initially, we clarified the choices of images amateurishly photographed by the researcher. It is understood that analytical geometry, in general, emphasizes the algebraic language, in which objects are identified by mathematical laws, which remains in line with calculus in teacher training courses. Geometric aspects related to such laws are rarely explored, as happens when addressing the concept of derivative. Only in specific cases, the study begins with the slope of a tangent line to a curve and arrives at the concept through a visual/geometric interpretation. Such an approach, in the opinion of this author, would be more significant than the application of derivation rules. Similarly, the concept of a definite integral is rarely obtained by visually looking at the arc length of a curve. Also, matrices could explore the visual aspects of symmetries.

For these reasons, exploring images such as the ones in Figure 1 can be motivating, since such an approach allows the visualization of surfaces approximate to the paraboloid and torus. In addition, such images are rich in elements of both spatial and plane geometry, and their constructions in dynamic software can foster many other concepts that can be motivating for the student and for the teacher in training.

As there was a wide possibility of answers/justifications, we chose two categories a posteriori: flat figures and spatial figures.

a) The first picture

Initially, of the twenty-nine respondents in the first group, only one indicated that they did not see a geometric shape in the figure, while all those in the second group responded that they had seen one, although the views, in both groups, were not always well characterized from the point of view of geometric conceptual view. Participant $PC_{26}$ made the following record: “no veo figuras geométricas[2]” (I don't see geometric figures). The incidence in visualized flat figures surpassed the spatial ones (Chart 1).

Chart 1 – Figures visualized by the first group

| Flat figures | Espace figures |
|---|---|
| Oval (8) | Sphere, Sheroid (5) |
| Circular shape, circle, círculo (5) | Ellipsoid intent (1) |
| Rectangle, quadrilateral, parallelogram (14) | Prism (1) |
| Ellips (7) | -- |

Source: Survey data (2021)[3].

The data in Table 1 indicate that there were thirty-four views of flat geometric figures in relation to seven spatial ones. Here, the importance of imagery cited by Hadamard (2009) is perceived, in the sense that a connection between image and language is necessary, be it verbal or written. In teaching, this connection is fundamental in order to develop geometric knowledge to register with words when observing a visual model. Apparently, the participants did not realize that these were representations of space objects.

Some justifications deserve a closer look, since in intuitionism, according to Poincaré (1969), it is not minds that have changed, but intuitive ideas. See what was mentioned by $PC_{26}$ about not seeing any geometric shape in the image provided for analysis.



Although it was said to be the images of bushes, some of the participants visualized shapes around them, such as:

#$PC_{29}$: “Rectangle, where the shadow of the tree is located, oval, the crown of the tree, in the background are identified some rectangles that are the buildings”.

#$PC_{24}$: “The base of the tree has a quadrilateral shape”.

#$PC_{20}$: “A sphere because it looks like a circle in the plane of the photo”. The justification presented indicates that its author has the perception of space representation, that is, he perceives the environment and his imagination leads him to formalism, connecting the object with its representation on the plane based on the assumptions of Piaget and Inhelder (1993).

#$PC_{18}$: “a rectangle on the ground and a circle on the tree”. It is noticed that the individual analyzed the whole in the image provided and not just the tree. However, it is not explicit what he indicates to visualize as a circumference. See that the rectangle is in the ground plane, so, in fact, it is a flat figure, in the same way as a circle. However, the circumference that he claims to see in the tree does not seem to have any meaning.

#$PC_{17}$: "Ellipse tree shape, straight white floor line, rectangle leaf path on floor around tree". The individual also analyzes the whole when he perceives the boundary line between the sidewalk and the street. Confuses the shape of the tree by claiming to be an ellipse, as it is a spatial object. Perhaps what you mean by the rectangle being the path of leaves on the floor is the rectangular space left without a floor. The statement that the leaves of the tree on the ground form an ellipse is also indicated by PC16.

The following rationale is interesting in that it flags flat and spatial objects.

#$PC_{10}$: “I identify 3D shapes and 2D shapes: segment, quadrilateral, prism, oval”. It does not explain what it sees as a prism.

In these records, it can be observed aspects that must be taken into account when teaching mathematics (KANDEMIR; GÜR, 2007), that is, not exploring algorithmic approaches, but new, current teaching methods, for example, imagination and creativity as o Brainstoring, providing students with a wealth of possibilities for visual approaches.

In what follows, the responses obtained in the second group for the same photograph are analyzed, summarizing them in Chart 2.

Chart 2 – Figures viewed by the second group

| Flat figures | Space figures espaciais |
|---|---|
| Half an ellipse (2) | Cone and truncated cone (3) |
| Parabola a (3) | Paraboloid (2) |
| Quadrilateral (1) | Elliptical paraboloid (1) |
| --- | Hemispheres (1) |
| --- | Half sphere (1) |
| --- | Semi-ellipsoid (1) |

Source: Survey data (2021)[3].



It can be seen, from Tables 1 and 2, that the diversification in plane figures of the second group is much smaller than that of the first group, while that of the spatial ones is greater, including with more elaborate geometric shapes, not always explored in courses of geometry in higher education. Some justifications deserve a different look, such as the following.

#$PB_1$: “Half of an ellipse, as if I had cut the bush in half. Or a parable, considering only the extreme”. It is noted that this individual did not mentally imagine or visualize as Leivas (2009) indicates for the last skill. In turn, he perceives, when considering only the extreme, that he is referring to the boundary of the object.

#$PB_6$: “The shape is similar to a truncated cone, but rounded at what would be the top base.” Here, there is some confusion carried out by the individual, since the generatrices of a conical surface are straight lines and the base is a circumference, but neither of the two elements is cited, since it states: 'rounded'.

#$PB_8$: “2d – makes me think of the parabola/ 3d- paraboloid”. This individual uses his intuition and imagination citing the two possibilities. It explores the representation of the object (on the plane) to indicate parabolas, however, visualizing the object in space. Correctly marks resembling the paraboloid, as will be seen later in this article, with the use of GeoGebra.

#$PB_9$: “Assuming it to be a spatial shape, I consider that the shape looks like a semiellipsoid, as it looks like half of the shape represented in the previous question”. This participant evoked the first question of the investigation, which consisted of the image of a tree similar to the ellipsoid, which is not analyzed in this article.

#$PB_{11}$: “Half of an ellipse rotated around its axis, top of the tree”. Although the writing was not precise, nor which curve would be rotated, this individual brought an important element, namely, obtaining surfaces by rotating plane curves.

The answers/justifications of this second group refer to what Kandemir and Gür (2007) pointed out about different forms of creativity in teacher training, that is, the imagination that becomes fluent in the present research when individuals need to observe a photograph and imagine a connection with geometric representations of objects in space.

By detailing the records of research participants through the observation of photographed images, Moreira (2011) is in line with the sciences regarding approaches and analysis of holistic and qualitative information, in this case, on geometric visual elements. Thus, when establishing percentage comparisons between the two groups, it is found that, in the first, approximately 121% (it should be noted that each individual could indicate more than one figure) of visualizations of flat figures appears in comparison to 43% of the group. second group. With regard to visualizations of spatial objects, there are 25% in the first group and 64% in the second group. This illustrates a considerable discrepancy between the two groups. Probably, such a difference occurs because, in the first group, diverse individuals participated in terms of institutions, although they chose to carry out the workshop that would address geometry. Meanwhile, in the second group, everyone participates in a specific group on teaching and research in Geometry. As for the academic training of both, there is no discrepancy.

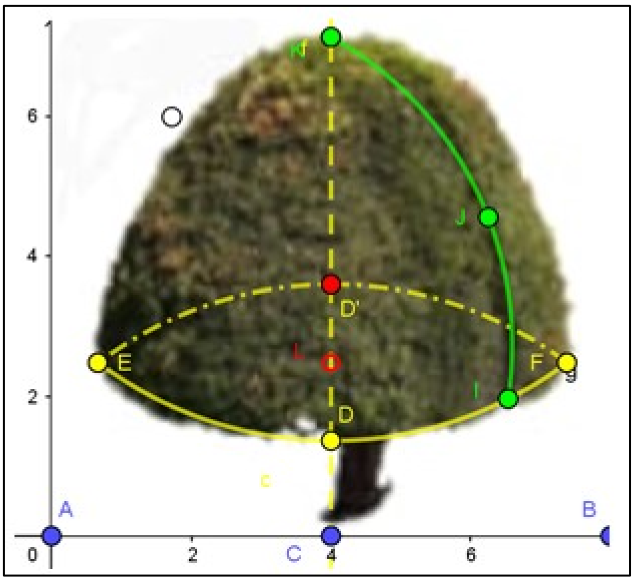
Let's see, next, how to explore a Dynamic Geometry software to approximate the rovided image with its formal representation (for the researcher, it resembles a paraboloid). The initial image is considered and, later, elements around the tree are removed, leaving only the same one, as shown in Figure 2.

Figure 2 – Images of the analyzed object

Source: Researcher's archive (2021).

The image on the right is taken to GeoGebra, as illustrated in Figure 3.

Figure 3 – Images of the object in GeoGebra.




Source: Own authorship (2021)[4].

The image is brought into GeoGebra by the 'insert image ... file' tool. It can be adjusted so that it is articulated at points A and B on the horizontal axis. The midpoint of AB, (C) is obtained and, through this, a perpendicular to the horizontal axis that meets the image in D at the bottom and in K at the top, as if it were through the interior of the spatial object. Explore a line parallel to the horizontal axis that meets the leftmost point of the image (E) and the rightmost point (F). An arc is drawn through E, D, F and then its reflection in relation to that straight line. Also, the symmetric of point D (D') with respect to the same straight line is obtained. With this, we have a representation of the circumference (approximate) with center L, which is the lower limit of the surface of the spatial figure represented there.

Still in Figure 2 (left), one can observe a certain flaw on the surface, which, in GeoGebra, was demarcated in green, that is, one can imagine, exploring the geometer's intuition, that there is the frontal part of a parabola with vertex K and concave towards the interior of the surface. The surface of the image is intuited as being obtained by rotating this parabola (generatrix), around the axis and resting on the circumference (directrix). This type of elaboration can help the teacher in the development of activities/contents of a geometry that is not always trivial, especially those involved in analytical geometry in which the equations, in general, are defined a priori, little providing the development of geometric thinking.

It is understood that, with such a study, it is possible to arrive at the concept of what paraboloid (of revolution) is, as defined by Lehmann (1970) as "Surface of revolution is the surface generated by the rotation of a given plane curve, around of a straight line fixed in the plane of the referred curve" (p. 363).

This realization or modeling in GeoGebra is not mathematically exact, however, the constructions can become useful for the study of quadric surfaces, which could encourage analytical geometry to explore the curves that can generate the surface (generatrices - IJK). The axis of revolution of the surface is the straight line, in this case, parallel to the vertical axis passing through points K and L. One could also explore the plane sections parallel to the circumference of the base (the parallels).

b) The second image

For this article, a second image was chosen, which the researcher associated with the surface of the torus. This surface can be observed, for example, in an air chamber of bicycles, automobiles, etc. The surface is very little explored in higher education disciplines, even though there are real prototypes. In the classic book of Analytical Geometry by Lehmann (1970), it appears in the proposed exercise as follows:

> Derive the equation of the surface of revolution generated by rotating the circle $x^2+y^2-2by+b^2-a^2=0$ and $z=0$ around the x axis. Construct the surface for a=2 and b=3. When b>a, the surface is called i, torus or anchor ring (LEHMANN, 1970, p. 367).

In the approach given by differential geometry, in a similar way, Valladares (1978) approaches the surface in parametric form as follows:

$$f(u,v)=((a+r\cos u)\cos v,\ (a+r\cos u)\operatorname{sen} v,\ r\operatorname{sen} u).$$

The numbers a and r are real, a>r >0, being the surface obtained by rotating the circle g(u)=(rcosu + a, o, rsenu) with center (a,0,0) and axis OZ.

Initially, from group 1, 10.3% answered that they did not see a geometric shape. Again, as in the previous image, the answers led to the establishment of two categories: flat geometric figures and spatial geometric figures. In Chart 3, these two categories are summarized.

Chart 3 -Synthesis of the figures visualized by the first group

| Flat figures | Space figures |
|---|---|
| Circumference and circle (6) | Torus, torus (7) |
| ellipses (2) | donut (4) |
| Circular sector (2) | |
| Girth rings, ring, hoop (3) | |

| | |
|---|---|
| straight (1) | |
| --- | |

Source: Survey data (2021)[3].

It is noticed that there was a certain reduction in the variation of geometric elements pointed out, with the visualization of flat images still being relevant. It is noted that the association of the spatial image associated with the torus geometric element was very pronounced, which caught the researcher's attention, since it is not usually used so much. Interesting that they associated the image with a 'doughnut'. For example:

#$PC_4$: "It looks like a donut, but I don't remember the geometric name". It appears that the individual had certainly already had contact with such a geometric object, however, he did not keep its name.

#$PC_5$: "A log, donut... the grass forms a ring". It is noticed that the participant meets what was indicated by Meissner (2015) when elaborating his mental process in order to seek the nature of the visualized image. Thus, it integrates, naturally, its intuition to the creativity, when providing the three possibilities of visualizing the image in real situations.

#$PC_{11}$: "Straight, the white line of the floor". Here, it is related to what Hadamard (2009) said that the participant, faced with a visual model, has many ways of thinking about a model that is presented to him, in this case, a bush. Thus, he tries to describe his intuitive perception focused on the straight line on the ground that surrounds the bush. Perhaps, on another occasion, he was not presented with the torus, whereas the straight line is more usual.

#$PC_{23}$: "The bushes look like ellipses, one inscribed inside the other". The visual image that the participant elaborates of the bush is neither strengthened by his imagination (being a spatial object), nor by creativity, as he could express how others elaborated, indicating that it resembles a donut, for example.

Next, we present the answers and justifications presented by the second investigated group.

Chart 4 – Summary of the figures visualized by the second group

| **Flat figures** | **Space figures** |
|---|---|
| Circumference (4) | Spatial (1) |
| Circular crown (3) | Torus (6) |
| Circumference rings, ring, hoop (3) | Hollow sphere (1) |
| Straight (1) | Donut (1) |
| | Part of a cylinder (1) |

Source: Survey data (2021)[3].

Analyzing the justifications of this group, it is verified that there is a smaller variation of geometric elements in relation to the one presented by the first group, both in terms of flat figures and spatial ones. In addition, the answers were approximate, with the exception of one individual who expressed having visualized a straight line, one who stated that he was part of a cylinder and one who identified a hollow sphere. Also, the justifications were more detailed, as can be seen in the ones presented below.

#$PB_2$: “I visualize a crown, for example, a wreath. But, geometrically, I can say that it could be a circle or a circular crown”. The participant, perhaps because it is a bush, associates the image with a wreath of flowers, which would not have the geometric meaning in the situation of the investigation. Then, he evokes geometry and indicates that it could be a circle (in reality, this visualization is not explicit) or a circular crown, since they are not flat geometric figures, but spatial ones.

#$PB_5$: “The bushes around the bed form part of a cylinder, seen as a cut parallel to the base”. Apparently, this individual does not perceive the curved line that forms the image and, to be a cylinder, the lines should be straight lines or segments of straight lines (generatrix).

#$PB_8$: “2d – circumference/3d- torus”. The participant makes his visual perception clear when interpreting the representation, which can be two- or three-dimensional.

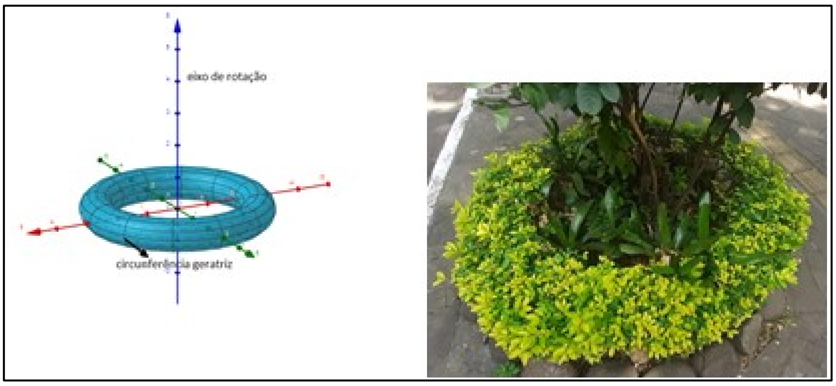
#$PB_9$: "The light green leaves, taken as a spatial figure, look like a torus to me, as they form something similar to an 'air chamber' ". The individual's record is interesting, as it explores his imagination and creativity by connecting the image with something very concrete, which, in general, is the prototype that illustrates the surface of the torus whenever this surface is evoked in texts. Its justification is in line with what the author of the article characterizes regarding visualization as a mental construct.

#$PB_{12}$: “Pingos de Ouro (*Duranta repens* L.) plantation is shaped like a log”.

#$PB_{14}$: “Circumference inside and outside the wheel”. Although he does not mention that it is similar to the torus, he notices the existence of two circles. It is seen that the individual does not claim to represent a torus, but a similarity with such a surface.

In order to approximate the captured image to the geometric shape of the torus of revolution, its representation in GeoGebra is considered, which is done from the equation of this surface. Type the following command in the input window: Surface ((R + r cos(v)) cos(u), (R + r cos(v)) sin(u), r sin(v), u, 0 , 2π, v, 0, 2π) (Figure 4).

Figure 4 – Torus obtained in GeoGebra through its equation and the representation in photo.




Source: Research data (2021)[4].

In Figure 4 (left), one can see a circle in a plane parallel to XOZ, the OZ axis, in addition to a circle in the XOY plane. Rotating the first around the OZ axis, resting on the circumference in the plane, we have the surface of the torus of revolution. With this, the photographed image of the bush (right) is approximated to the geometric surface.

The two images, placed side by side, reinforce the visual intuitive aspects of the researcher by characterizing visualization as a process of forming mental images in order to communicate certain concepts, in this case, a torus formed by bushes observed around a tree in a sidewalk. In addition, the meaning of intuition elaborated by Davis and Hersh (1995), which is the opposite of rigorous, although it is not, as such, very accurate, is registered here. Also, it is plausible or convincing, in the absence of proofs, what the visuals presented in GeoGebra soften. Finally, it relies on a physical model or on important examples, that is, almost heuristic.

## FINAL CONSIDERATIONS

In this article, geometric visualization is approached from representations in photographs of some ornate shrubs on sidewalks and urban gardens. In visual observations, intuition and imagination are evoked, as well as creativity. The research investigated two distinct groups, one consisting of participants in a workshop at an event outside Brazil, with 29 participants, and the other with Brazilian members of a geometry study and research group. The investigation aimed to analyze how respondents visualize geometric shapes found in bushes in an urban environment and associate them with spatial geometric shapes. Both groups responded to a Google Forms.

For this article, the answers and justifications of two of the six images provided that resemble a paraboloid and a torus were analyzed. When analyzing the answers provided, they were classified into two categories: flat figures and spatial figures. Although representations of bushes, thus real-world space objects, were presented, all of them, in reality, consisted of space elements. However, it was observed that there was a huge designation of flat geometric figures, which, in the researcher's opinion, characterizes a strong tendency in the teaching of these geometric elements in favor of spatial and even other geometries, as indicated by previous research from the author.

Visualizing the geometry involved around individuals, whether by intuition, imagination or creativity, seems to be relevant to the acquisition of geometric thinking throughout schooling. This seems to be important for the confrontation/training of several areas of knowledge, such as Engineering, Architecture, etc.

A close look at research involving visualization, in particular, indicates a certain timidity in international publications. However, it is thought that there are a variety of possibilities to articulate such skills analyzed here, which can be worked on, in particular, in the training of mathematics teachers or even in the training of pedagogues who will initiate the geometric formation of individuals. When arriving, for example, at calculus courses, there may be a better use in this discipline if students are able to visually interpret problems such as derivation or integration, not simply by applying formulas.

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Revista Brasileira de Ensino de Ciência e Tecnologia

# VISUALIZANDO FORMAS GEOMÉTRICAS EM ARBUSTOS


## RESUMO

Neste artigo, apresenta-se uma pesquisa qualitativa, a qual teve o objetivo de analisar como dois grupos distintos visualizavam formas geométricas constantes em registros figurais de arbustos encontrados no ambiente urbano de uma cidade da região metropolitana da capital gaúcha. Constam, desta análise, duas das seis imagens investigadas, as quais se aproximam de um paraboloide e de um toro. O primeiro grupo teve 29 participantes de uma oficina ministrada pelo autor em um evento internacional realizado na Colômbia, com estudantes e/ou professores de várias nacionalidades e níveis de formação. O segundo grupo, com 14 participantes, todos brasileiros e membros de um grupo de estudos e pesquisas liderado pelo autor. Os resultados mostraram limitações dos participantes em visualizarem formas geométricas, especialmente, as espaciais. Grande parte deles identificou mais objetos geométricos planos do que propriamente as duas formas geométricas espaciais e, poucos, identificaram o paraboloide e o toro.


**PALAVRAS-CHAVE:** Paraboloide. Toro. Percepção visual. Ambiente urbano.

## NOTE

1. The Dicio dictionary, available at https://www.dicio.com.br/imagistica/. defines imagery as a faculty or power of imagination, invention or fantasy.

2. From now on, citations will be presented translated by the author into English.

3. Organized by the author.

4. Built on Geogebra.

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**Received:** Jun. 22nd, 2022.
**Approved:** Nov. 14th, 2022.
**DOI:** 10.3895/rbect.v15n3.15641
**How to cite:** LEIVAS, J. C. P. Visualizing geometric shapes in bushes. **Brazilian Journal of Science Teaching and Technology**, Ponta Grossa, Special Edition, p. 1-18, Dec. 2022. Available at: <https://periodicos.utfpr.edu.br/rbect/article/view/15641>. Access on: XXX.
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