Revista Brasileira de Ensino de Ciência e Tecnologia
ISSN: 1982-873X

https://periodicos.utfpr.edu.br/rbect

A contextualized approach to mathematics in engineering: the potentialities of effective teacher questioning

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ABSTRACT

This article aims to present the results of two out of the three remote meetings of an intervention in which notions related to the real functions of a real variable were addressed through a problem in the context of the study of the characteristic curve of a semiconductor diode, created and implemented according to the theoretical and methodological framework of the theory of Mathematics in the Context of Sciences. Data were constituted by audio and video recording of the meetings. Seven first-term students majoring in control and automation engineering voluntarily participated in the research. The investigation focused on the communication established between the researchers and their research participants about the types of questions asked by the researchers, the answers given by the students and to what such communication brought to light. Attention was paid to aspects related to the transposition of mathematical knowledge into a field of application and the mobilization of mathematical and general competencies that constitute the epistemological basis of engineering. The results suggest that the competencies of handling symbols, representing mathematical entities, and communicating in, with and about mathematics are to be better developed by the participants. Regarding the general competencies that constitute the epistemological basis of engineering, the data showed that the ability to conduct research in the literature, use databases and other sources of information, as well as work with situations which requires critical assessment in order to draw conclusions should be better explored. The most noticeable cognitive difficulty related to the mathematical object function was working with the different representations of a function in an articulated manner. In terms of epistemological obstacles, we point to the belief held by students that the order of variables in a function is irrelevant. In relation to the transposition of mathematical knowledge to their fields of application, students demonstrated to face difficulties in working with the symbology related to functions, which may result, among other causes, from the fact that there are more than two magnitudes involved in the algebraic expression of functions. Finally, the research participants evaluate that experiencing the process of solving that problem provided them with the opportunity to expand their views on the need to exercise the application of mathematics in solving real problems and to better understand mathematics and physics concepts.

KEYWORDS: Contextualized Event. Functions. Diode. Teacher Questioning. Competencies.

INTRODUCTION[1]

One of the themes that has been explored by the members of the Mathematical Education in Higher Education Working Group (GT-04) of the Brazilian Society of Mathematical Education is the teaching and learning of mathematics in courses to which this science is at service, with engineering occupying a prominent role in this scenario. As one can find in Gomes, Bianchini and Lima (2021), based on Bernhard (2015), Christensen *et al.* (2015) and Christensen and Mejlgaard (2015), some of the pillars of reflections in the field of engineering education have been: the understanding of the fields of knowledge fundamental for the training and professional practice of the future engineer and how to actively engage undergraduate students in their own learning processes using didactic-pedagogical strategies, such as problem-based learning.

Christensen and Mejlgaard (2015) shed light to the five major areas of research in Engineering Education, namely:

1. **Engineering Epistemologies:** research on what constitutes engineering thinking and knowledge within social contexts now and in the future.
2. **Engineering Learning Mechanisms:** research on engineering learners' developing knowledge and competencies in context.
3. **Engineering Learning Systems:** research on the instructional culture, institutional infrastructure, and epistemology of engineering educators.
4. **Engineering Diversity and Inclusiveness:** research on how diverse human talents contribute solutions to the social and global challenges and relevance of this profession.
5. **Engineering Assessment:** research on, and the development of, assessment methods, instruments, and metrics to inform engineering education practice and learning.

This research focuses on two of these fronts: the mechanisms and learning systems in engineering. Thus, the present article firstly emphasizes the main results of the research at hand, thoroughly discussed in a previous publication (see Gomes, Bianchini e Lima [2021]), which concern the first of three meetings held with first-term engineering students. During these meetings, notions considering the real functions of a real variable were addressed through a problem related to control and automation engineering and related degrees, which was situated in the context of the study of the characteristic curve of a semiconductor diode. Secondly, using the same methods and criteria, it analyzes the events occurred in the second meeting.

The problem was developed and implemented according to the theoretical and methodological framework of the theory of Mathematics in the Context of Sciences. The analysis points to the interaction between the authors of this article (who implemented the problem) and their students, research subjects, focusing on the types of questions asked, the answers given, and the aspects revealed by such answers. The goal was to analyze **the communication between the actors participating in the resolution of a mathematical problem contextualized in engineering specific situations and what it reveals when one considers students' knowledge, their skills in transposing it from mathematics to analog electronics,**

the obstacles faced in the transposition process, and the mathematical competencies mobilized by the students.

THEORETICAL FRAMEWORK

To investigate the teaching and learning of mathematics in engineering courses, the authors have adopted the theory of Mathematics in the Context of Sciences (henceforth TMCS). This framework has been developed by researcher Patricia Camarena Gallardo at the National Polytechnic Institute of Mexico for almost four decades to support reflections on mathematics education in higher education, especially in programs that do not hold mathematics as their main object of study, but as a science at the service of the training of some professionals. Considering engineering courses, Camarena (2015, p.112) highlights that, within this theoretical framework, mathematics is conceived as a "tool and language of engineering". Besides, it has a formative character, for the development of "a mathematical culture and mathematical thinking enables the student to act in society in an informed, critical, analytical, and scientific way" (CAMARENA, 2017, p. 2).

According to TMCS, from a historical-epistemological perspective, scientific knowledge has been developed in an integrated manner. Therefore, the framework assumes that university programs that draw from mathematics are supposed to train professionals capable of transferring mathematical knowledge to the areas that require it (CAMARENA, 2013). In this sense, three elements become essential: interdisciplinarity, contextualization, and contextualized transposition. These are articulated in TMCS through the notion of contextualized events (CE), i.e. problems, projects or case studies contextualized in situations that can be derived: (i) from the other classes that make up the engineering curriculum (these being the most appropriate for basic classes, such as mathematics); (ii) from the professional activities that students will exercise after graduating, or (iii) from their daily lives (CAMARENA, 2017).

When proposed in mathematical classes, CE allow the relation between mathematical and non-mathematical classes to occur, especially engineering core classes. This way, CE function as instruments for contextualizing mathematics from an interdisciplinary perspective, with a view to providing students with opportunities for developing skills to perform a contextualized transposition. Such process, according to Camarena (2004), is understood as the adjustments necessary for some mathematical knowledge taught in a specific class to become a piece of knowledge to be applied in specific engineering situations.

The notion of contextualized transposition, developed in TMCS, aligns with Buch and Bucciarelli (2015, p. 499) when they criticize the idea that knowledge is "something that individuals can instrumentally put to use – regardless of context". As Bernhard (2015) points out, introducing professional adequacy in engineering programs is highly desirable, which means to provide engineering students with problems they will likely find in their work as engineers. Besides offering the potential to meet this goal, the work with CE is deemed essential to allow students to exercise and master contextualized transposition. However, the creation of events is a key and non-trivial task for the professional who teaches mathematics in engineering courses.

One of the difficulties often found in this task lies in the **contextualization-decontextualization paradox**, as Christensen *et al.* (2015) would put it: while engineers employ mathematical knowledge in particular contexts and work in a context-sensitive manner, mathematicians and scientists from basic sciences tend to use the concepts inherent in their fields of knowledge in a decontextualized fashion, often displaying some difficulty in recognizing the ways in which they can be contextualized. Furthermore, as the authors point out, context cannot be "an end in itself, but rather a means to a certain end" (CHRISTENSEN *et al*., 2015, p. xxiii). When adopting TMCS, the goal is to make students understand, through context, why they should learn a certain mathematical theme or concept, in which aspects it enables them to meet the demands of their future profession, and to mobilize or develop mathematical competencies.

A competency, according to Camarena (2015, p. 118), "is the cognitive mobilization of the attributes of a professional to face a problem situation making use of the integration of all their knowledge, skills, attitudes, and values". Regarding specific mathematics competencies, we bring Niss (2003) to the fore. To him, mathematical competence "means the ability to understand, judge, do, and use mathematics in a variety of intra-and extra-mathematical contexts and situations in which mathematics plays or could play a role" (NISS, 2003, p. 7). According to Niss (2003), these are the mathematical competencies: (C1) thinking mathematically (mastering mathematical modes of thought); (C2) posing and solving mathematical problems; (C3) modelling mathematically (analyzing and building models); (C4) reasoning mathematically; (C5) representing mathematical entities; (C6) handling mathematical symbols and formalisms; (C7) communicating in, with, and about mathematics; and (C8) making use of aids and tools, IT included.

Besides developing these competencies, when searching for a context and, consequently, elaborating a CE, the teacher should consider the cognitive difficulties and epistemological obstacles that may be faced and should be, as far as possible, minimized (BROUSSEAU, 1983). Taking these aspects into account, Camarena and González (2001) suggest a sequence of steps aimed at data collection that will substantiate the construction of a CE. These steps include analysis of books of core engineering classes, books and teaching plans of mathematical classes, a historical-epistemological study of the mathematical object that one wishes to address, and the difficulties of cognitive nature that, based on research carried out by other researchers, are related to such object. This was the strategy used for the construction of the CE which was implemented and will be analyzed in this article. For further information on the process of preparing this CE, read Lima, Bianchini and Gomes (2021).

For the event at hand, a didactic-pedagogical organization that allows the future engineer the development of general competencies is proposed. These, according to Grimson and Murphy (2015), are constituted by three layers (L1, L2, and L3) that form the epistemological basis of engineering: (L1) the competent use of knowledge built before entering the university; (L2) competencies linked to those that should be the results, in terms of learning, of an engineering course; and (L3) competencies required of the engineering professional. Linked to TMCS, the didactic model of mathematics in context corroborates this concern, since it centers the student who, in collaborative work teams constituted by members with different learning styles, will build mathematical knowledge from the joint resolution of the CE (CAMARENA, 2017).

The implementation of the event took place in three meetings of two hours each. In this article, we firstly shed light to the main results of the analysis of the first meeting (Gomes, Bianchini and Lima, 2021) and using the same standards previously adopted, we then analyze the second meeting. The analysis focuses on: 1) the types of questions proposed by the teachers, also authors of the article, both the ones previously created and those that naturally emerged during the two meetings, 2) their objectives, and 3) what could be understood through students' responses in relation to their abilities to carry out the contextualized transposition of the mathematical notions at hand, their mathematical and general competencies, the epistemological obstacles, and the cognitive difficulties they faced.

The discussion is then linked to the communication in which teachers and students engage in class, since, corroborating with Botelho and Rocha (2015), we understand it to be a primary factor for the mathematics teaching and learning processes to work well. In this article, the term mathematical communication is assumed to be **the ability to express mathematical ideas or notions, a powerful mediator of change in complex cognitive behaviors, for it can promote meaning-making, self-monitoring and reflection, as well as the co-construction of new ideas** (MARLIANI; WALUYA; CAHYONO, 2021; NATHAN; KIM, 2009).

To Machado *et al.* (2020), there are four discursive actions to be mobilized to promote communication: explain, question, listen, and respond. In this article, we focus on the practices of **questioning** and consequently of **listening** when analyzing students' responses, provided that, as Diaz *et al.* (2013) argue, a fundamental educational goal is for teachers to create spaces in which students can development their thinking. Considering communication, teachers are supposed to use appropriate strategies of asking questions, in the sense of “encouraging, extending and, more importantly, challenging students' thinking” (DIAZ *et al.*, 2013, p. 163). Machado and Lacerda (2020, p. 2) see the act of questioning as essential in communication, given that when one creates a question, “fertile ground is established for understanding something. This way, questions play a very important role in the organization of mathematical tasks".

In accordance with Tienken, Goldberg and Dirocco (2009, p. 40), when posing questions, teachers should be aware that, from the perspective of the cognitive demand required by students when answering a question, there are differences between productive questions —"which provide students with the opportunity to create, analyze, or assess”—and reproductive questions—"which stimulate students to imitate, recall, or apply knowledge and information taught by the teacher through a simulation process”. The authors contend that “teachers need to plan a route and strategy to use questions productively and develop students' thinking based on their classes’ learning objectives” (TIENKEN; GOLDBERG; DIROCCO, 2009, p. 42).

Yenmez *et al.* (2018), in their turn, argue that the purposes teachers have when asking questions shape the types of questions they will ask, since these questions aim to “verify students' knowledge or guide students' thinking, focus students' attention on different mathematical strategies, or lead them to explain or justify their thoughts” (p. 2).

Based on Fazio (2019), this investigation assumes three main overlapping categories of questions that may be asked by a teacher to promote learning: those that require **retrieval** (aiming at eliciting prior knowledge), those that require **metacognition** (demanding some kind of reflection on the reasoning adopted) and those that involve **reasoning** (asking to deduce something from one or more premises).

In the didactic organization proposed for the CE, an a priori series of **guiding questions** was designed, to **guide students in their use of mathematical concepts and procedures to solve problems** (SAHIN; KULM, 2008). Such questions, which should be answered by the students during the meetings, can be considered **factual**, because they required a predetermined response and allowed researchers to identify the basic knowledge of students and how these were transposed into the context of engineering.

In addition to these questions, a series of other questions were proposed to the students during the intervention. These were deemed as **competent questions** (VISEU; OLIVEIRA, 2012) once we sought to listen to students' responses to collect data that would allow us to infer about their reasoning processes. The factual questions proposed were classified according to the typology conceived by Boaler and Brodie (2004) shown in Table 1.

Table 1 - Question types and their respective descriptions

Question type	Description
T1. Gathering information, leading students through a method	Requires immediate answer. Rehearses known facts/procedures. Enables students to state facts/procedures.
T2. Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them.
T3. Exploring mathematical meanings and/or relationships	Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations.
T4. Probing, getting students to explain their thinking	Asks students to articulate, elaborate, or clarify ideas.
T5. Generating discussion	Solicits contributions from other members of class.
T6. Linking and applying	Points to relationships among mathematical ideas. Points to relationships among mathematics and other areas of study/life.
T7. Extending thinking	Extends the situation under discussion to other situations where similar ideas may be used.
T8. Orienting and focusing	Helps students to focus on key elements or aspects of the situation in order to enable problem-solving.
T9. Establishing context	Talks about issues outside of math in order to enable links to be made with mathematics.

Source: Adapted from Boaler and Brodie (2004, p. 777).

We now proceed to explain the CE, its didactic organization and the methodology used to have it implemented.

THE CE, ITS DIDACTIC ORGANIZATION, IMPLEMENTATION PROCEDURES AND RESEARCH METHODOLOGY

In a first moment, we, researchers and authors of this text, decided to create a CE concerning the study of aspects of the theory regarding semiconductor diodes through one of the steps proposed by Camarena and González (2001). Then, a situation related to the study of the characteristic curve of a semiconductor diode was identified in the book **Electronic devices and circuit theory**, by Boylestad and Nashelsky (2013), making it possible to approach the real exponential functions of a real variable contextualized in the field of control and automation engineering. Although this is a pilot study, the CE presented below is thought to be appropriate to be studied in a class of Differential and Integral Calculus, so that novice students can revisit the content, now directed to engineering, not as a review of what they studied in high school.

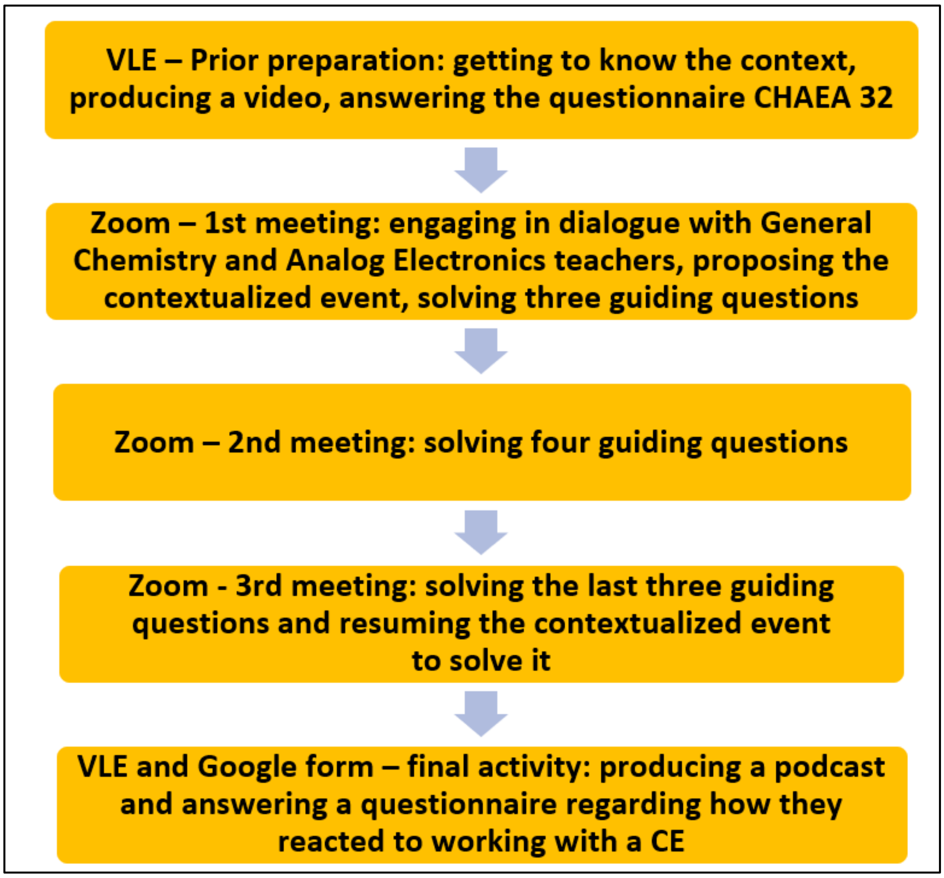
Contextualized Event: a diode, like all other electronic components, needs a certain amount of time to go from a conducting to a non-conducting state; this is called the diode recovery time. Many practical applications require diodes that "recover" easily, i.e. that go from its conducting to its non-conducting state in the smallest possible time interval. One of the silicon diodes that features this characteristic is 1N4148, one of the diodes most used in electronics and that has a recovery time of 4 nA. The *datasheet* of diode 1n4148, which highlights its electrical characteristics, can be found at https://pdf1.alldatasheet.com/datasheet-pdf/view/551820/WINNERJOIN/1N4148.html.

By studying concepts related to solid-state physics, it is suggested that the general characteristics of a semiconductor diode can be defined by the following equation, referred to as Shockley´s equation, for the forward- and reverse-bias regions: $I_F = I_R\left(e^{\frac{V_F}{nV_T}} - 1\right)$. In this equation, I_F is the direct current across the diode, I_R is the reverse saturation current, V_F is the applied forward-bias voltage across the diode, n is an ideality factor, which is a function of the operating conditions and physical construction, and V_T is the thermal voltage, defined by: $V_T = \frac{kT_K}{q}$ where k is the Boltzmann constant whose value is $1{,}38 \times 10^{-23}$ J/K, T_K is the absolute temperature in Kelvin, which is given by adding 273 to the temperature in Celsius, and q is the magnitude of electronic charge, which is given by $1{,}6 \times 10^{-19}\,C$.

Considering the information presented above and assuming that the diode 1n4148 is subjected to a current of 30 mA, determine the forward voltage drop across the diode and the approximate values of its saturation currents at the following temperatures: – $\boldsymbol{45°C}$, $\boldsymbol{50°C}$ and $\boldsymbol{125°C}$.

Didactically, the implementation of the CE (Figure 1) was organized in the following way: prior preparation (carried out in a virtual learning environment (VLE) asynchronously), and three two-hour synchronous meetings (via Zoom platform), in which 10 guiding questions were asked. At the end of the third meeting, the event was effectively solved and, at a later time (asynchronously), the students carried out two last activities to complete the activity.

Figure 1- Steps for implementing the CE



Source: The authors (2022).

Seven first-term students majoring in control and automation engineering in a private institution in the state of São Paulo, Brazil, voluntarily participated in the implementation of the CE conducted by the authors of this article. Data were composed by the videos produced by the students at the time of prior preparation, their written production during the synchronous meetings, the audio and video recordings of such meetings, the podcasts prepared by the students at the end of the activity, and the questionnaire they answered after the end of the CE implementation.

This article sheds light on the main results of the first meeting and details the analysis of the second meeting, focusing on the data obtained through the audio and video recording of these moments, which involve students´ answers to the guiding questions proposed by their teachers and their responses to these questions. During the analyzed meetings, the students were divided in two groups, one with three and another with four members. All students who expressed themselves orally or in writing were considered in the analysis.

In the meetings under discussion, while both groups of students answered the guiding questions previously elaborated, the teachers/researchers would propose sub-questions formulated during the intervention based on students´ reactions, which included their doubts, comments, misunderstandings about what was being studied or even in the face of their inertia before the guiding questions. In the following section, a synthesis of the most relevant aspects observed from the analyses related to the first meeting is presented.

MAIN RESULTS FROM THE FIRST MEETING

In the first meeting, as shown in Figure 1, students worked with the first three guiding questions and related sub-questions presented in Table 2. To have readers better understand this research, the acronyms used in Tables 4, 5, 6, 7, and 8 are now explained. Ti refers to the typologies of questions (BOALER; BRODIE, 2004) presented in Table 1; $Si.j$ refers to the sub-question j related to the guiding question i; and $RSi.jx$ indicates the replies to the sub-question j related to the guiding question i given by a student x.

Table 2 - Guiding questions and related sub-questions asked in the 1st meeting

Guiding Question 1: Can Shockley´s equation explain a functional relationship? If so, what is the dependent variable and what is the independent variable?
Sub-questions related to guiding question 1
S1. 1: What is/characterizes a functional relationship?
S1. 2 Is any relationship occurring between two variables considered a function? What happens to the elements in the domain and the elements in the image in a functional relationship? In which ways are they related?
Guiding Question 2: Is thermal voltage a function of any variable? Explain. If your answer is affirmative, represent this function using a graph.
Sub-questions related to guiding question 2
S2. 1: (When observing the students entering the thermal voltage function in GeoGebra, one of the researchers asked) You switched T_K, which in the expression that gives the thermal voltage is given by $T + 273$, where T is the temperature in Celsius, by x only. Where, then, can we find you adding 273 to the temperature in Celsius? Suppose you want to find the image at a specific point in the domain of the function you entered in the input field of GeoGebra. For example, if the temperature is 25° Celsius, what value will you assign to x?
S2. 2: Why is the function representing thermal voltage increasing and represented by a line?
S2. 3: Upon hearing one student stating that to build "*a graphical representation of the thermal voltage function one just needs to draw a straight line, no matter the slope*", one of the researchers asked: considering the equation that represents thermal voltage, what is the angular coefficient?
S2. 4: (Asking the group who did not use GeoGebra) Why didn't you create the graph using GeoGebra?
S2. 5: Why do you say that the line which represents the thermal voltage starts at the origin?
S2. 6 Question: Why can't the temperature be negative? How come there is no zero temperature? I don't understand!
Guiding Question 3: Given that the diode 1n4148 operates between -65℃ and 175℃, determine the range of variation of the thermal voltage of this diode in such interval.
Sub-questions related to the guiding question 3

S3. 1: Now that you have answered question 2, what should be done to solve question 3?
S3.2: What is the lowest limit for the temperature being considered here?
S3.3: Isn´t it the value of x that you want it to be -65?

Source: The authors (2022).

Considering the answers the students gave to the questions and sub-questions presented in Table 2, the following aspects were identified:

- the ability to conduct research in the literature and use databases and other sources of information (E2), essential to the engineer, should be better explored (GRIMSON; MURPHY, 2015);
- when transposing the concept of function from mathematics to one of its areas of application, in this case analog electronics, students face difficulties in working with the symbology surrounding it. This can be explained by the fact that only two magnitudes are represented, mostly by x and y, in most situations studied in Calculus classes, whereas when studying functions, there are more than two magnitudes in the algebraic expression to represent them;
- some students seem to deem the order of variables as irrelevant when working with functions;
- students find it hard to differentiate dependent and independent variables and to distinguish the latter from constants and the independent term of an algebraic expression;
- students also demonstrate certain difficulty in discriminating between the action **obtain f(a)** and the task **find the values of x for which $\boldsymbol{f(x) = a}$**;
- students also struggle to analyze different aspects of a function by looking at its graphical representation.

Furthermore, answering the questions and sub-questions has enabled students to mobilize different mathematical competencies (NISS, 2003) and skills related to the layers that make up the epistemological basis of engineering (GRIMSON; MURPHY, 2015):

- practicing the competency of **reasoning mathematically** (C4);
- critically assessing the information given (L2);
- developing the competency of **handling mathematical symbols** (C6), a competency which needs to be better worked according to our analysis;
- mobilizing the competency of **representing mathematical entities** (C5);
- triggering the competency of **thinking mathematically** (C1);
- exercising the work with algebraic and graphical representations of functions through software and, as a result, fostering the competency to **use instruments and tools**, IT included (C8).

By answering guiding question 3, students were able to exercise the ability to apply their knowledge and understanding to solve engineering problems using established methods (L2 and L3 for Grimson and Murphy [2015]). Furthermore, students also needed to transpose (CAMARENA, 2004) a mathematical procedure to the context of the thermal voltage range of a diode (image set) according to the given temperature range (domain set) in which the diode works. Finally, they were able to exercise their ability to identify, locate, and obtain required data, assess them critically and, thus, draw conclusions (L2).

PRESENTATION AND ANALYSIS OF DATA REGARDING THE SECOND MEETING

At the second meeting, four guiding questions were proposed (see Table 3).

Table 3 - Guiding questions asked in the 2nd meeting

Questions
Q4. Considering the information given in the *datasheet* of diode 1n4148, answer: (i) what is its reverse saturation current (I_R) at 25°C at a reverse-bias voltage (V_R) of $20\ V$? (ii) As presented in the *datasheet*, given that the diode 1n4148 generally requires a forward voltage (V_F) of $0{,}86\ V$ to conduct a direct current (I_F) of $10\ \mathrm{mA}$, determine its ideality factor.
Q5. Considering the diode 1n4148, construct a graphical representation for I_F in function of V_F considering a temperature of $25°C$.
Q6. Analyzing the graphical representation built in question 5, answer: (i) what happens to the values of V_F as the values of I_F increase unlimitedly? How could such behavior be explained using the algebraic expression of I_F? (ii) what happens to the values of I_F as the values of V_F decrease unlimitedly? How could such behavior be explained using the algebraic expression of I_F? (iii) in the graphical representation built in question 5, the first quadrant represents the diode forward-bias region. Can you observe, in this region, a point where there is a change in the behavior of I_F? If so, what is this point and what is its significance in the context of the study of diodes? (iv) what is the current conducted when the forward voltage is $0{,}86\ V$? Was this behavior expected? Explain. (v) in the graphical representation built in question 5, the third quadrant represents the diode reverse-bias region. In this region, what is the meaning of working with negative current values and negative voltage values? From a physical point of view, are such values really negative? (vi) Describe the behavior of I_F in function of V_F in the reverse-bias region (3rd quadrant).
Q7. From your answers to question 6, by what algebraic expression could you approximate Shockley´s equation in the diode forward-bias region? And in the reverse-bias region?

Source: The authors (2022).

The analyses call attention to the guiding questions and sub-questions that generated discussions concerning mathematical concepts. The questions asked throughout the second meeting, as one can read, are at various times more related to the physical context, which allowed reflections on aspects of analog electronics. It seems important to underline this fact to highlight the relevance for mathematics teachers to commit themselves to self-training when choosing this contextualized approach in order to build basic knowledge about the extra mathematical context with which they will have to deal.

The initial analysis, presented in Table 4, refers to item (ii) of guiding question 4.

Table 4 - Analysis of guiding question 4 (ii) and its sub-questions, according to Boaler and Brodie´s (2004) typologies

Guiding question 4 (ii): As presented in datasheet, to conduct a direct current (I_F) of 10 mA, the diode 1n4148 requires, in general, a forward voltage (V_F) of 0.86 V. Determine its ideality factor.		
Factual	**Requires retrieval and reasoning**	**Includes aspects of typologies T2, T3, T5, T6 and T9**
Objective: Taking into account the interpretation of the problem, the information obtained in item (i), which asked students to determine the reverse saturation current (I_R) of diode at 25°C at a reverse bias voltage (V_R) of 20 V, and the substitution of the known values of I_F, I_R, V_F and V_T in Shockley's equation, solve an exponential equation and determine the value of the ideality factor n.		
Sub-questions and answers given by students		
S4ii. 1: What is the I_R at 25° C?		
Factual	**Requires retrieval**	**Includes aspects of typologies T1 and T8**
(When the researcher does not get an answer to the proposed question, understanding the difficulty the student is facing, he guides the participant to analyze item i of Question 4, reinforcing typology T8). **RS4ii. 1a**: Does V_T , which is the thermal voltage, correspond to 25°? (The researcher replies that the thermal voltage depends on the 25°C, as seen in the first meeting and, after understanding what was V_T, the student goes back to analyze Shockley's equation and asks the following question) **RS4ii. 1a:** Is e the...? A researcher confirms that e is the base of the neperian logarithm, that it is a constant, an irrational number.		
S4ii. 2: How can you determine n? What kind of mathematical operation or mathematical object will you need to work with to get the value of n?		
Factual	**Requires retrieval, metacognition, and reasoning**	**Includes aspects of typologies T1, T2, T3, T4, T5 and T8**
RS4ii.2b: We will use logarithm because we are solving an exponential equation.		
S4ii. 3: What if I modify the value of V_T or I_R? Would you have to do all the calculations again, starting from scratch? Has anyone felt like isolating n in Shockley's equation, without directly substituting the data?		
Factual	**Requires reasoning**	**Includes aspects of typologies T1, T2, T3, T4, T5 and T7**
RS4ii. 3a: I only realized I could have done this when I was at the end of the calculation.		

Source: The authors (2022).

When seeking strategies to solve guiding question 4 (ii), a student replies with another question (**RS4ii. 1**), revealing difficulties related to the L1 of the epistemological base of engineering (GRIMSON; MURPHY, 2015). On basic knowledge of mathematics, he doubts whether the number e in Shockley's equation is, in fact, the same e already studied by him. As for basic knowledge of physics, he incorrectly takes a measure of voltage as a measure of temperature.

Such obstacles hinder his ability to understand what magnitudes are involved in Shockley´s equation, which of them have their magnitudes known and, consequently, what is the best way to determine what is still unknown. It seems that the student failed to grasp the reason why the item (i) of the question was proposed when, in fact, the goal was to get him to obtain one of the elements that he should use in (ii).

Additionally, it should be noted that the procedure adopted by the students (when they directly replace all known values in Shockley´s equation and make the calculations to obtain the ideality factor, motivating the researcher´s questioning **S4ii. 3**) points to their inability to seek more economical strategies before performing the calculations, which may indicate that teachers are disinvested in teaching such possibility. These strategies would allow the establishment of general relationships between known and unknown variables and could be more directly applied in similar situations. In other words, these are procedures that would help students obtain the ideality factor of any diode from any given conditions.

With regard to the layers presented in the epistemological base of engineering (GRIMSON; MURPHY, 2015), we argue that responding to the guiding question and its sub-questions (see Table 4) enabled students: to employ physical and mathematical background knowledge acquired before entering higher education (L1); apply their knowledge and understanding to plan the solution of a problem involving other areas with which they had not yet encountered; identify, locate and obtain required data (L2); integrate knowledge from different areas (in this case, mathematics, physics and analog electronics) and levels of complexity (L2); and appropriately apply methods to analyze and solve engineering problems (in this case, determine the ideality factor of a semiconductor diode) (L3).

In relation to mathematical competencies (NISS, 2003), answering questions allowed students to think and reason mathematically (C1 and C4), solve a mathematical problem (in this case, an exponential equation) (C2), analyze a mathematical model (Shockley's equation and how it allows obtaining the ideality factor of a diode) (C3), handle mathematical symbols (C6) and use technological tools (students resorted to Excel and GeoGebra) (C8).

The analysis of guiding question 5 is shown in Table 5.

Table 5 - Analysis of guiding question 5 and its sub-questions, according to Boaler and Brodie´s (2004) typologies

<table>
<tr><th colspan="3">Guiding question 5: Considering the diode 1n4148, design a graphical representation for I_F in function of V_F considering a temperature of 25°C.</th></tr>
<tr><td>Factual</td><td>Requires retrieval and reasoning</td><td>Includes aspects of typologies T1, T2, T3, T4, T5, T6 and T9</td></tr>
<tr><td colspan="3">Objective: To provide an opportunity for students to transpose the idea of graphical representation of a function from mathematics to analog electronics.</td></tr>
<tr><td colspan="3">Sub-questions and answers given by students</td></tr>
<tr><td colspan="3">Noticing that one of the students constructed the graphical representation of the function given algebraically by $f(x) = e^x$ and claimed to have answered question 5, the researchers ask:</td></tr>
</table>

S5. 1: Why have you concluded that the graphical representation of I_F in function of V_F considering a temperature of $25°C$ coincides with the graphical representation of $f(x) = e^x$? Why, in your opinion, the function that translates the relationship between I_F and V_F is e^x?		
Factual	**Requires metacognition and reasoning**	**Includes aspects of typologies T3, T4, T5 and T8**
RS5. 1a: I know that the scale I used is not right. **RS5.1b:** This (pointing to the exponent of the number e in Shockley´s equation, i.e., $\frac{V_F}{nV_T}$) is what can change (that is, what is variable, according to him).		
S5. 2: Consider Shockley´s equation and compare it with the function f whose algebraic expression is $f(x) = e^x$.		
Factual	**Requires retrieval, metacognition, and reasoning**	**Includes aspects of typologies T1, T3, T4, T5, T6 and T8**
RS5. 1b: Oh, it is not the $f(x) = e^x$ itself. The function (algebraically represented by Shockley´s equation) has undergone some transformations (with respect to $f(x) = e^x$) to become $I_F = I_R\left(e^{\frac{V_F}{nV_T}} - 1\right)$.		

Source: The authors (2022).

Both the answers given by the students and the questions posed by the researchers are described in Table 5. When students identify the function whose algebraic representation is given by Shockley´s equation with the exponential function given by $f(x) = e^x$, the research subjects, in our analysis, demonstrate some difficulty associated with the misperception that the graph of a function is a geometric model of a functional relationship, with no need to be reliable to it. The same happens when they confirm this identification by reporting to the researchers that the scale was incorrect and that the exponent was the only element that varied in that equation. Such difficulty is also felt when students work with different representations of a function (in this case the algebraic and the graphic representations), when they display fragmented knowledge in relation to the study of functions, and when they lack the ability to grasp them as abstract and high-level mathematical objects.

With regard to the competencies linked to the layers of the epistemological base of engineering (GRIMSON; MURPHY, 2015), students firstly reveal weaknesses about the notion of exponential function (L1), not taking into account that there will be times in which they will not work with the most elementary function ($f(x) = e^x$), a fact that may push students to recognize the possible transformations that it can undergo giving rise to other functions such as the one represented algebraically by $I_F = I_R\left(e^{\frac{V_F}{nV_T}} - 1\right)$. Later, however, after talking to the researchers, students recognized such transformations. The analysis also indicates that teachers should prioritize work with situations that allow students to critically assess the circumstances and, as a consequence, draw informed conclusions (L2).

In terms of mathematical competencies (NISS, 2003), the question and the sub-questions presented in Table 5 provided the opportunity to: think and reason mathematically (C1 and C4), solve a mathematical problem (construct the graphical representation of a function) (C2), analyze a mathematical model (translated by Shockley's equation) (C3), represent mathematical entities (in this case, graphically represent the relationship between the forward-bias voltage and the forward-bias current in a semiconductor diode) (C5), communicate in mathematics (through the sketch of the graph requested) (C7), and use technological tools (in this case, GeoGebra, to build the graph) (C8).

Table 6 - Analysis of the guiding question 6i and its sub-questions, according to Boaler and Brodie´s (2004) typologies

<table>
<tr><th colspan="3">Guiding question 6i: What happens to the values of V_F as the values of I_F grow limitlessly? How could such behavior be explained using the algebraic expression of I_F?</th></tr>
<tr><td>Factual</td><td>Requires retrieval, metacognition, and reasoning</td><td>Includes aspects of typologies T1, T2, T3, T4, T5, T6 and T8</td></tr>
<tr><td colspan="3">Objective: Analyze the behavior of a function algebraically given by Shockley´s equation from the observation of its graphical and algebraic representations.</td></tr>
<tr><td colspan="3">Sub-questions and answers given by students</td></tr>
<tr><td colspan="3">Upon hearing one of the students answer the question affirming that it tends to infinity, one of the researchers asked:
S6i.1: Who tends to infinity?</td></tr>
<tr><td>Factual</td><td>Requires reasoning</td><td>Includes aspects of typologies T1, T2, T3, T4, T5, T6 and T8</td></tr>
<tr><td colspan="3">RS6i.1b: The value of V_F which is on the y-axis ... because V_F is in the exponent.
After hearing the student, one researcher states: However, V_F is on the x-axis.
Surprised, the student reacts: Oh, is it?</td></tr>
<tr><td colspan="3">S6i.2: What happens to the variations between the values of V_F when the values of I_F increase too quickly?</td></tr>
<tr><td>Factual</td><td>Requires retrieval, metacognition, and reasoning</td><td>Includes aspects of typologies T1, T2, T3, T4, T5, T6 and T8</td></tr>
<tr><td colspan="3">RS6i.2a: As I_F increases limitlessly, the variations in the values of V_F get smaller and smaller.</td></tr>
<tr><td colspan="3">S6i. 3: How can the behavior you noticed be deduced from the algebraic expression of the function? If we did not have the graphical representation of the function, could we understand its behavior from its algebraic expression?</td></tr>
<tr><td>Factual</td><td>Requires retrieval, metacognition, and reasoning</td><td>Includes aspects of typologies T1, T2, T3, T4, T5, T6 and T8</td></tr>
<tr><td colspan="3">RS6i.3b: I think so, and this is because of the logarithm. When the exponent increases too fast, so does the exponential.</td></tr>
</table>

Source: The authors (2022).

Response **RS6i.1b** suggests that, given the articulation of the algebraic and graphical representations of the function given by Shockley´s equation, the student may have struggled to identify both the dependent and the independent variables and, consequently, to distinguish the values of which magnitudes are being indicated on each of the coordinate axes. This perception is reinforced by the surprise shown by the student to the answer given by one of the researchers that the values of V_F are indicated on the abscissa axis. This very answer also points to a common, fix idea, often reinforced by the more colloquial language adopted by teachers in their classes, that the ordinate is always represented by y and that therefore the terms ordinate and y-axis are, in any situation, synonymous. Such idea also reveals an aspect that can constitute an obstacle to the effectiveness of contextualized transposition (CAMARENA, 2004) of mathematics to a context of application (such as analog electronics), in this case, the transposition of the concept of function and its graphical representation.

In turn, the response **RS6i.3b** indicates that the student sees exponential and logarithmic functions as apparently equivalent. Although these functions are usually addressed at the same time in a class when teachers define one in terms of the other, they are two distinct mathematical objects. In the case presented in Table 6, no explicit mention of the logarithm can be found in the expression being studied by the students. Therefore, the justification to be given is equally not related to the logarithm, but to the exponential function.

Table 7 - Analysis of guiding question 6ii and its sub-questions, according to Boaler and Brodie´s (2004) typologies

Guiding question 6ii: What happens to the values of I_F as the values of V_F decrease limitlessly? How could such behavior be explained using the algebraic expression of I_F?		
Factual	**Requires retrieval, metacognition, and reasoning**	**Includes aspects of typologies T1, T2, T3, T4, T5, T6 and T8**
Objective: Analyze the behavior of a function algebraically given by Shockley´s equation from the observation of its graphical and algebraic representations.		
Sub-questions and answers given by students		
Upon hearing from one of the students, in response to question 6ii, that *as the values of V_F decrease limitlessly, the values of I_F tend to zero, but do not reach zero*, and from another student that *this indeed happens and that there is a horizontal asymptote* ($y = 0$), the researcher asks: **S6ii. 1:** How come I_F will never be zero? Analyze the algebraic expression of Shockley's equation. What is the value of I_F when V_F is equal to zero?		
Factual	**Requires retrieval and reasoning**	**Includes aspects of typologies T1, T2, T3, T4, T5, T6 and T8**
RS6ii. 1c: Oh, when V_F is equal to zero, then I_F will also be zero (the student replies in awe).		
The students then go on to more carefully analyze the scale with which the graphical representation of the function represented algebraically by Shockley´s equation was being presented in GeoGebra. When noticing this concern, one of the researchers asks: **S6ii. 2**: Observe the graph and check: in the reverse-bias region, is the graphical representation of the function described by Shockley´s equation the horizontal line $y = 0$?		
Factual	**Requires retrieval and reasoning**	**Includes aspects of typologies T1, T3, T4, T5, T6 and T8**
RS6ii. 2a: No, it's a little lower.		
S6ii.3: How much lower? How can you know this by analyzing the d*atasheet*?		
Factual	**Requires retrieval, metacognition, and reasoning**	**Includes aspects of typologies T1, T3, T4, T5, T6, T8 and T9**
After adjusting the scale with which the graph was being viewed, one student answers: **RS6ii. 3b**: As the values of V_F decrease, the graphical representation will approach a horizontal line, the horizontal asymptote I_R.		
S6ii. 4: What is the behavior of the exponential term of Shockley´s equation when the values of V_F decrease limitlessly?		

Factual	Requires retrieval and reasoning	Includes aspects of typologies T1, T3, T4, T5, T6, and T8
RS6ii. 4c: The exponents of e, namely $\frac{V_F}{nV_T}$, become negative and then we have $\frac{1}{e}$ raised to the exponent position with a positive sign. Therefore the exponential term of Shockley's equation, which is $I_F = I_R\left(e^{\frac{V_F}{nV_T}} - 1\right)$, approaches zero. Thus, I_F will become the same as $-I_R$.		

Source: The authors (2022).

The answers given to the researchers' questions, also presented in Table 7, reinforce the previous comment that the ordinate axis and the y-axis are taken as synonymous. In fact, in order to help students acknowledge the behavior of the dependent variable as the values of the independent variable decrease unlimitedly in the situation at hand, the researchers themselves resort to an abuse of notation when they refer to the axis of the dependent variable I_F as y-axis – practice that should be done very carefully to avoid potentializing existing obstacles (see **S6ii. 2**). The data presented in Table 7 also show that students do not seem to spontaneously articulate the graphical and algebraic representations of a function. The scale used in the graphical representation being displayed in the software (GeoGebra) was not suitable for visualizing the behavior of the function for V_F less than or equal to zero. This way, had students not seen this problem nor considered the algebraic expression of the function articulated to its graph, they would have concluded that the values of I_F would never be null or negative. This is exactly what happened.

Table 8 - Analysis of the guiding question 6iv and its sub-questions, according to Boaler and Brodie´s (2004) typologies

Guiding question 6iv: What is the current conducted when the forward voltage is $0,86\ V$? Was this behavior expected? Explain.		
Factual	**Requires retrieval, metacognition, and reasoning**	**Includes aspects of typologies T1, T3, T4, T5, T6, T8 and T9**
Objective: To determine, in the context of the study of a diode, the image of a function for a given element of the domain.		
Sub-questions and answers given by students		
After a student replied to a question using a graphical analysis to locate the abscissa point 0.86 on the curve which is the graphical representation of the function, and obtaining the ordinate of such point, one of the researchers asks: **S6iv. 4:** Could you reach this result in another way, perhaps working with the algebraic expression of the function and considering the concept of image?		
Factual	**Requires retrieval and reasoning**	**Includes aspects of typologies T1, T2, T3, T4, T5, T6, T8 and T9**
RS6iv. 1b: (the student replies the researcher with another question) Do you mean if I could find this 0.86? **RS6iv. 2a:** No, we already have 0.86. We now have to put the 0.86 in the V_F.		

Source: The authors (2022).

Response **RS6iv. 1b** gestures once again towards some difficulty in distinguishing the values given from those that should be determined. More specifically, the students are not yet fully able to specifically discriminate the values assumed by the dependent variable of a function from those assumed by the independent variable.

In regards to the answers given to the aspects analyzed in guiding question 6 and the sub-questions linked to it, the data presented in Tables 6, 7 and 8 denote some difficulties related to mathematical aspects that should have been constructed before entering the university (L1, according to Grimson and Murphy [2015]), especially the ability to articulate the different representations of a function, to distinguish dependent and independent variables, and to identify functions that, although related to one another and taught most of the time together, are distinct, such as the case of exponential and logarithmic functions. These difficulties suggest the urgency to foster the development of competencies 5 and 7, which are respectively: representing mathematical entities and communicate in, with and about mathematics, according to Niss (2003).

Considering aspects related to the epistemological basis of engineering (GRIMSON; MURPHY, 2015), the action of answering guiding question 6 and the questions that emerged from them also encouraged teachers to shed light on the multidisciplinary context of engineering, and fostered, in some cases, the construction of knowledge associated with engineering programs learning outcomes (L2): the scientific and mathematical principles relevant to both control and automation engineering and related degrees, as well as key concepts of these areas, the ability to solve problems they had not yet encountered, the ability to identify, collect, and critically analyze data, and the ability to integrate knowledge from different areas and levels of complexity. In addition to this result, the research subjects were able to develop, albeit initially, competencies required of professional engineers (L3): the appropriate application of methods to analyze and solve engineering problems and the effective use of interpersonal and communication skills.

In terms of mathematical competencies (NISS, 2003), the work with the questions and sub-questions mentioned in Tables 6, 7 and 8 allowed the mobilization and ongoing process of developing the eight aforementioned competencies.

FINAL REMARKS

In this article, we analyzed two of three remote meetings that aimed to provide first-term engineering students, our research participants, with the opportunity to solve a problem. To do so, students were supposed to activate their mathematical knowledge related to the real functions of a real variable contextualized in analog electronics, specifically dealing with the study of the characteristic curve of a semiconductor diode. This investigation sought to analyze the communication between the actors participating in the resolution of a mathematical problem contextualized in engineering specific situations and what it reveals when one considers students ' knowledge, their skills in transposing it from mathematics to analog electronics, the obstacles faced in the transposition process, and the mathematical competencies activated by the students.

For the analytical work, the types of questions (BOALER; BRODIE, 2004) proposed by the authors of this article, which included both previously elaborated questions and those that emerged naturally throughout the two meetings, were considered. In addition, the objectives of such questions as well as our understandings concerning students´ abilities to carry out the contextualized transposition (CAMARENA, 2004) of the mathematical notions being worked, their mathematical (NISS, 2003) and general (GRIMSON; MURPHY, 2015) epistemological and cognitive difficulties (LIMA; BIANCHINI; GOMES, 2021) were equally emphasized.

The guiding questions and sub-questions proposed by the researchers throughout the activities allowed students to work and apply, at different times, the eight mathematical competencies suggested by Niss (2003). Data analysis suggested that those competencies related to (i) handling symbols, (ii) representing mathematical entities and (iii) communicating in, with and about mathematics still need to be better developed by the research participants, which demands that teachers direct their actions to promote these competencies.

In relation to the general competencies that constitute the epistemological basis of engineering (GRIMSON; MURPHY, 2015), the data indicate students´ difficulties in recovering and mastering basic knowledge of mathematics and physics acquired prior to their entrance in the university. They also indicate that the ability to conduct research in the literature and use databases and other sources of information, essential for a professional engineer, must be better explored. Yet another result points to the need to deepen students´ learning, which requires active and ongoing work with situations that demand a critical assessment of what is given so as students are capable of drawing conclusions.

The answers provided by the students point to some cognitive difficulties related to the notion of function, fact widely discussed in the area, as highlighted by Lima, Bianchini and Gomes (2021). In a nutshell, some of our findings suggest that students have difficulties in: (i) identifying the ordered pair composed by an element of the domain and its respective image for functions given in algebraic form when the value of the image is provided and the corresponding value of the pre-image (or inverse image) is sought, (ii) working with the different representations of a function in an articulated way, especially establishing dialogues between its algebraic and graphic representations, as well as analyzing different aspects of a function given its graph, (iii) handling the symbology related to the concept of function, especially when having to discriminate between the action of obtaining $f(a)$ and the task of finding the values of x for which $f(x) = a$ distinguishing both the dependent variable from the independent variable and the independent variable from constants and from the independent term of an algebraic expression, (v) seeing functions as high-level abstract objects, and (vi) identifying functions that, although linked and taught most of the time together, are distinct mathematical objects (such as, for example, the exponential and logarithmic functions).

In terms of the epistemological obstacles related to the notion of function, the analyses direct our attention to two of them, which are identified by Sierpinska (1992), namely: (i) deeming the order of variables as irrelevant and (ii) thinking that the graph of a function is a geometric model of a functional relationship, which does not have to be reliable and can contain points (x, y) such that the function is not defined in x.

With regards to the contextualized transposition (CAMARENA, 2004) of mathematical knowledge to its fields of application, in this case, transposing the concept of function from mathematics to analog electronics, we contend that the aforementioned difficulties in working with the symbology related to functions arise from the fact that there are more than two magnitudes involved in the algebraic expression at hand (Shockley´s equation), unlike what occurs in most situations dealt with in Calculus, in which there are only two magnitudes more frequently represented, regardless of context, by the letters x and y. The practice of abuse of notation has as one of its consequences the formation of a solidified idea that the ordinate axis is always represented by y, which may constitute an obstacle to the graphical representation of a functional relationship in a context of application.

The work with contextualized events such as the one discussed in this article encourages students ' reflection and dialogue between them, their peers and teachers; therefore, it opens spaces for the discussion on fundamental issues related to the mathematical object at hand or notions associated with it. The data also stress one of the questions asked regarding a very sensitive issue from a mathematical point of view, but oftentimes not treated with due importance and proper care in Calculus classes: whether, when the values of a function tend to infinity, we can or cannot affirm that this function has a limit. During the meeting in which this discussion was brought to light, the researchers explained that it was a complex situation, because at the same time we hear that for the limit of a function to exist it must be a real number, we also hear in situations like the one being questioned that the limit is infinite. In other words, one may know the behavior of a certain function and that this behavior can be described as unlimited. However, stating that the limit of a function is infinite constitutes, in fact, an abuse of notation.

This article ends with some students' perceptions about the activity. These demonstrate that, although the work with contextualized events demands commitment and dedication from both teachers and students due to its challenging character and to requiring an ongoing overcoming of barriers of diverse nature, it is highly valued by those who participate in it. They claim to recognize, among other potentialities, (i) the opportunity to experience “a different approach to the use and teaching of mathematics related to other engineering classes, which gives another dimension to the physical and mathematical concepts that we regularly learn in the program”; (II) the possibility to “broaden our vision regarding the need to more fully exercise the application of mathematics in solving real problems” and (III) the need to “see beyond theory”. As a final remark, we invite other researchers to dedicate themselves to the elaboration, implementation, and analysis of contextualized events for the teaching of mathematics in engineering courses.

UMA ABORDAGEM CONTEXTUALIZADA DA MATEMÁTICA NA ENGENHARIA: AS POTENCIALIDADES DAS PERGUNTAS DOS PROFESSORES

RESUMO

Neste trabalho, tem-se por objetivo apresentar os resultados de dois dos três encontros remotos de uma intervenção na qual noções relacionadas às funções reais de uma variável real foram abordadas por meio de um problema no contexto do estudo da curva característica de um diodo semicondutor, elaborado e implementado de acordo com os preceitos teóricos e metodológicos da Teoria A Matemática no Contexto das Ciências. Por meio de dados obtidos da gravação em áudio e vídeo dos encontros, dos quais participaram, voluntariamente, sete estudantes ingressantes de um curso de Engenharia, com interesse na habilitação Controle e Automação, direcionou-se a atenção à comunicação estabelecida entre os pesquisadores e os sujeitos, no que se refere aos tipos de questionamentos feitos pelos pesquisadores, às respostas dadas pelos estudantes e aos aspectos por elas revelados. Atentou-se a aspectos relativos à transposição de conhecimentos da Matemática para um campo de aplicação e à mobilização de competências matemáticas e competências gerais que constituem a base epistemológica da Engenharia. Entre os resultados, observa-se que as questões norteadoras e as subquestões delas advindas, propostas pelos pesquisadores, possibilitaram concluir que as competências de manusear símbolos, representar entidades matemáticas e comunicar-se em, com e sobre a Matemática precisariam ser mais bem desenvolvidas pelos sujeitos. No que se refere às competências gerais que constituem a base epistemológica da Engenharia, nota-se que a habilidade para realizar pesquisas na literatura e usar bases de dados e outras fontes de informação deve ser mais bem explorada, assim como o trabalho com situações que requerem avaliação crítica do que é dado para elaborar conclusões. A dificuldade cognitiva mais perceptível relacionada ao objeto matemático função foi trabalhar, de maneira articulada, com as diferentes representações de uma função. Em termos de obstáculos epistemológicos destaca-se o de considerar a ordem das variáveis em uma função como irrelevante. Em relação à transposição de saberes matemáticos para seus campos de aplicação, evidencia-se que os estudantes participantes enfrentam dificuldades em trabalhar com a simbologia relacionada às funções, as quais são provenientes, entre outras causas, do fato de haver mais do que duas grandezas envolvidas na expressão algébrica com a qual se estava operando. Por fim, destaca-se que, na percepção dos sujeitos, vivenciar o processo de resolução do problema proposto oportunizou que ampliassem suas visões quanto à necessidade de exercitar a aplicação da Matemática na resolução de problemas reais e que compreendessem de outras formas os conceitos desta ciência e da Física.

PALAVRAS-CHAVE: Evento Contextualizado. Funções. Diodo. Questionamentos Docentes. Competências.

NOTE

1. This article is an expansion of the text presented and published in the conference proceedings of the VIII SIPEM, which took place in November 2021, and will be cited as Gomes, Bianchini and Lima (2021) throughout the article.

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Received: Jun. 17th, 2022.
Approved: Nov. 14th, 2022.
DOI: 10.3895/rbect.v15n3.15615
How to cite: BIANCHINI, B. L.; GOMES, E.; LIMA, G. L. A contextualized approach to mathematics in engineering: the potentialities of effective teacher questioning. **Brazilian Journal of Science Teaching and Technology**, Ponta Grossa, Special Edition, p. 1-28, Dec. 2022. Available at: <https://periodicos.utfpr.edu.br/rbect/article/view/15615>. Access on: XXX.
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