

Teaching concepts on fractal geometry to visually-impaired students: from case of study to teaching instrumentation

ABSTRACT

The intervention research reported here, of an applied nature and a qualitative approach, aims to analyze the contributions of material, tactile and descriptive adaptations, on Fractal Geometry teaching and learning for students with visual impairment, in order to enable them access school content on this topic. These subjects need adequate tools that allow them to access and appropriate mathematical scientific knowledge. In this way, a practical study was developed, in an educational service center specialized in the area of visual impairment, in a municipality in the Midwest of Paraná state, with a group of five blind and visually-impaired people, already graduated from High School. The collection of data from the pedagogical intervention happened through observations, film records and a field diary, examined through conversation analysis. The results indicated gaps in the students' education in relation to the mathematical concepts necessary for Fractal Geometry learning. The study found that the use of adapted materials for teaching concepts related to this content, such as tactile images, contributed to the formation of the concepts taught. The results point to the need to promote training for Mathematics teachers in the area of Inclusive Education, with a view to effective inclusion and appropriation of scientific knowledge by students with such a disability. Thus, the results obtained in this study corroborate the development of the project Rethinking Teacher Education: Foresting Inclusive Practices for Visually-Impaired Students in Mathematics Classes, sponsored by Columbia University of CO and Lemann Foundation.

KEYWORDS: Mathematics teaching. Visual impairment. Tactile adaptation. Fractal Geometry.

Lucia Virginia Mamcasz Viginheski
lmamcaszvgiginheski@gmail.com
[0000-0002-6474-0927](tel:0000-0002-6474-0927)
UniGuairacá, Guarapuava, Paraná, Brasil.

Sani de Carvalho Rutz da Silva
sani@utfpr.edu.br
[0000-0002-1548-5739](tel:0000-0002-1548-5739)
Universidade Tecnológica Federal do Paraná, Ponta Grossa, Paraná, Brasil.

Elsa Midori Shimazaki
emshmazaki@uem.br
[0000-0002-2225-5667](tel:0000-0002-2225-5667)
Universidade Estadual de Maringá, Maringá, Paraná, Brasil.

Marcio Pascoal Cassandre
mpcassandre@uem.br
[0000-0001-9415-4315](tel:0000-0001-9415-4315)
Universidade Estadual de Maringá, Maringá, Paraná, Brasil.

INTRODUCTON

The research proposal “Rethinking Teacher Education: Foresting Inclusive Practices for Visually-Impaired Students in Mathematics Classes”, promoted by Columbia University and the Lemann Foundation, through the Researching Teaching and Learning Project: An Imperative for Equity in Teacher Education teachers, was conceived from the premise of the right of all people to access, permanence and progression in school and the conception that there are problems in the process of inclusion of people with disabilities in school, among them, mainly in the discipline of Mathematics, for example, the scarcity of resources that allow access to knowledge through other channels of reception, different from vision; predominance of teaching through orality, among others.

Studies on teaching Mathematics to visually-impaired people have been developed in recent years (PEREIRA; BORGES, 2020; ALVARISTO; SILVA; VIGINHESKI; PILATTI, 2020; SILVEIRA; SÁ, 2019; VIGINHESKI; SILVA; SHIMAZAKI; PACHECO, 2017; PASQUARELLI; MANRIQUE, 2016; VIGINHESKI, SILVA; SHIMAZAKI; ANJOS, 2016; SGANZERLA; GUELLER, 2014; FERNANDES; HEALY, 2010; MANRIQUE; FERREIRA, 2010, FERNANDES, 2008). Despite the growing number of these researches and other technical-scientific advances, such as the access of people with disabilities to regular education, teachers still have difficulties in teaching them. Among these difficulties, the materials for teaching the different contents of the subject to people with disabilities are considered rare, demonstrating the need for instruments to be developed for mediation in the teaching and learning process.

Faced with this limitation, it is possible to resort to studies in the area of Mathematics developed with non-visually-impaired people, to make adaptations according to the needs of students with disabilities, among them, the studies developed by Paulino, Guilherme, Coelho Neto and Damin, 2018; Gervásio, 2017; Troian, Santos and Lima, 2017; Pereira and Oliveira, 2016; Lopes, Alves and Ferreira, 2015, Semmer, Silva, Neves and Pilatti, 2015, and others.

Viginheski, Aires, Silva, Pilatti, Frasson, and Shimazaki (2017) evaluated the feasibility of adapting some research developed in the area of Mathematics for teaching visually-impaired. One of them was developed by Nascimento (2012 a, b), when presenting a proposal for Fractal Geometry teaching, with the objective of proposing different teaching activities that allowed students to perceive the existence and basic characteristics of its concept. Thus, Nascimento (2012b) considers that the teacher, by making use of fundamentals related to other geometries, such as fractal, using different forms of teaching, enables students to participate in the teaching and learning process. And this participation provides visually impaired students with the appropriation of different mathematical concepts.

In this sense, for Talizina (2009), actions and operations by students on the object of knowledge represent the psychological mechanism of concepts, because, without them, the concept cannot be internalized, nor generalized in other situations.

Concepts, when elaborated by action, are established as a real and complex act of thought. Thus, they cannot be taught through training, repetition, or only through orality and/or lists of exercises, as is common in schools for the study of

Mathematics. According to Galperin (2009), for knowledge to be internalized and to promote development, some steps are necessary for the teaching process. Among them, the formation of action on the material or materialized plane. It is understood, in the case of visually-impaired students, to be necessary at this stage, the use of tactile adaptations to the visual materials used, in order to allow access to information through other sensory channels, other than vision, to enable the student to act on the object or its representation.

The second stage concerns the formation of action in terms of external language, in which language is the interaction between students, the teacher, and the object of study. Language allows the transposition to the mental plane through the operation with signs, characterizing the third stage. In it, external communication becomes internal language, allowing the student to perform the action mentally independently (GALPERIN, 2009).

Teaching, from this perspective, goes beyond what is done through the oral exposition of contents and repetition and, often, in the case of the presence of visually-impaired students in the classroom, it constitutes an excluding teaching, especially when the contents taught depend on visual representations, as in the case of Geometry.

Among the knowledge in the Geometry area, Fractal Geometry is highlighted. James Gleick, quoted by Nascimento (2012a, p. 5), considers that “for the mind's eye, a fractal is a way to glimpse the infinite”. From this, the question is: how is it possible to glimpse the infinite through tactile perceptions by blind students? As with the visual representation of fractals, in which there is a limit to visual representation and perception, but not to the mind, tactile representations are also limited; hence the need for the teacher to mediate, with the adapted material, in the appropriation of this knowledge by students with visual impairments, for the elaboration of the concept in the mental plane.

Viginheski, Aires, Silva, Pilatti, Frasson e Shimazaki (2017), based on the analysis of Nascimento's research (2012b), propose some possibilities for adapting tactile materials for teaching Fractal Geometry to blind and visually-impaired people. It is understood that it is necessary to perform and validate them, using them in the teaching of Mathematics for students with this disability. Thus, this study aims to analyze the contributions of material adaptations, tactile and descriptive, in the Fractal Geometry teaching and learning content for visually-impaired students.

Studies on Fractal Geometry are part of the common core of the curricula of schools in the state of Paraná (PARANÁ, 2008). However, there are still few studies on the subject. Considering that, the present study is justified and supported by the need to discuss the theme, based on the right that blind and visually-impaired students have to access all the contents that are taught at school, with the necessary modifications that meet their needs and also by the fact that the teaching of fractal geometry contributes to the development of students' logical thinking (KORNILOV, 2013), to expand the horizons of knowledge (NASCIMENTO, 2012b), among other contributions.

METHODS

The applied nature and qualitative approach research was developed in an educational service center specialized in the area of visual impairment, in a city located in the Midwest state of Paraná. At the time the research happened, the school served 135 students and offered specific support services for the visually-impaired students, such as Braille, Soroban, Orientation and Mobility, Autonomous Life Activity, Visual Stimulation, Art, Music and Physical Education.

Five male visually-impaired people participated in the study, whose ages ranged between 27 and 47 years. All the students had already completed High School, and one of them had a degree in Business Administration. Two were blind from birth and three became visually impaired as adults; one lost his sight completely and for the other two the loss was partial. Chart 1 presents a description of the study participants. It is noteworthy that the names are fictitious, to preserve their identity:

Chart 1 – Participants of the reasearch

| Name ¹ | Age | Education | Visual Impairment | | | |
|-------------------|-----|-------------|-------------------|-----------|------------|----------|
| | | | Low Vision | Blindness | Congenital | Acquired |
| Bruno | 40 | College | X | | | X |
| Carlos | 27 | High School | | X | X | |
| João | 26 | High School | | X | X | |
| Luiz | 47 | High School | X | | | X |
| Miguel | 43 | High School | | X | | X |

Source: CAEE-DV Secretary (2019).

The research has the approval of the Research Ethics Committee of the State University of Maringá - UEM. At the beginning, a meeting was held with the research participants, informing them of the objective, the procedures, as well as the need to sign the Free and Informed Consent Term - TCLE, as determined by research ethics.

The study, developed in five meetings lasting two hours each, included the teaching of: i) theoretical aspects of Fractal Geometry; ii) non-linear fractals (of nature) and iii) geometric fractals (Koch Curve; Sierpinski Triangle; Sierpinski carpet; Peano Curve).

The instruments used for data collection were observations, records through films and a field diary. The material collected was stored in a database with restricted access to researchers, for exclusive use for the analyses. The data were studied through conversation analysis, focusing on the context of the actions and their interpretations, determining the principles and mechanisms present in the actions and their reaction, reciprocally, between the interlocutors (FLICK, 2009).

For the introduction of the Fractal Geometry theoretical aspects, the existence of natural and artificial elements in the environment in which they live was orally

discussed with the students, which can be or cannot be represented by defined geometric shapes, that is, the Fractal Geometry characteristics and history.

In that same meeting, the blind students used touch to explore some objects brought by the researcher, which show the characteristics of nature's fractals, such as vegetables, including broccoli and different types of leaves, as well as some species of flowers and others. With the students who lost their sight throughout their lives, the characteristics of fractals present or not in various elements of nature, such as clouds, trees, lightning, plants, buds, shells, among others, were discussed.

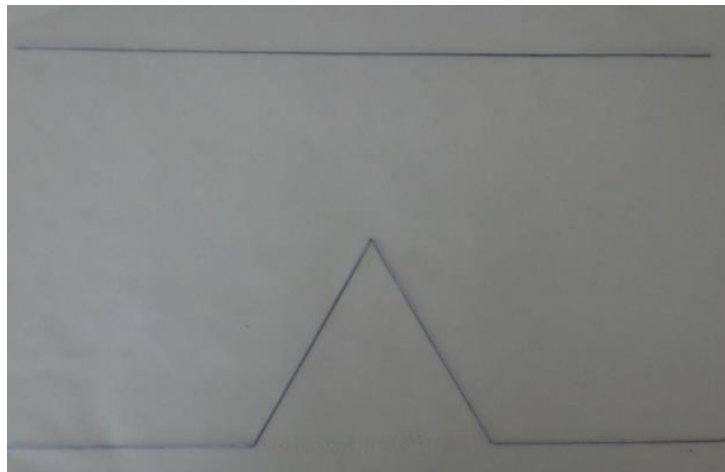
The students studied geometric fractals from the exploration of the Koch Curve fractal, for which concepts of line segment, equilateral triangle, length measurement were explored, in addition to proceeding with the systematization of the fractal length, through measurements of successive iterations. For the Sierpinski Triangle fractal, the concepts of midpoint, perimeter, and area of triangles were studied together with the representation through the power of the number of triangles at the n level of iterations. For the Sierpinski Carpet, the concepts of square, parallel lines, perpendicular lines, perimeter, and area were studied. In three levels of iteration, the relationship between the side of the square, its perimeter, and its area was established.

The materials used in the teaching and learning process were adapted according to studies by Valente (2010, 2008); Reily (2004) and Spider (2000). For Valente (2008), there are two ways of adapting images for visually-impaired people: tactile adaptation and adaptation through description.

In tactile adaptations, the images are made with materials that create reliefs, among these materials are lines, string, embossed glue, embossed thermoforming in plastic, clay, wood, embossed points, among others. In adaptations through description, the images are portrayed through texts, which need to be objective, brief and efficiently convey the details necessary for the elaboration of concepts, in the case of Mathematics (FIORINI; MANZINI, 2010; VALENTE, 2008).

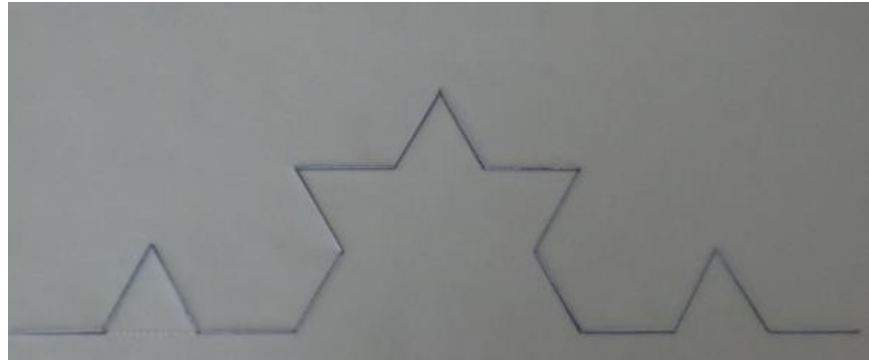
For the present study, in the tactile adaptations, embossed and textures in the representations were used, such as suede paper and tracing paper. Figures 1 to 6 show the adaptations made:

Figure 1 – Level 0 and Level 1 of the Koch Curve



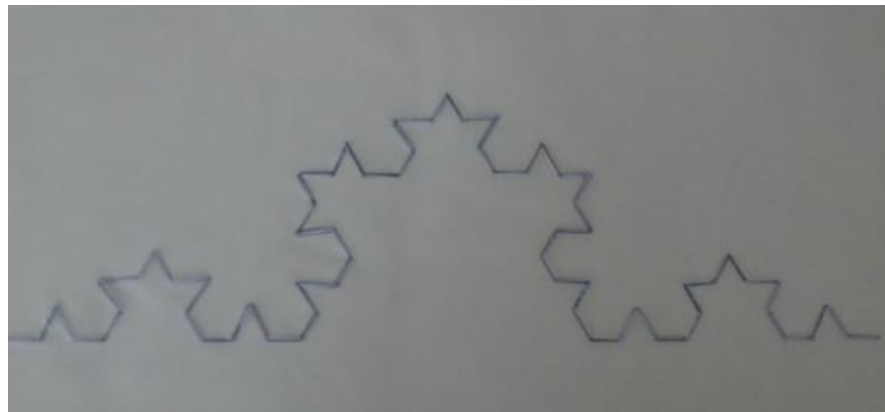
Source: Researchers' file (2019).

Figure 2 – Level 2 of the Koch Curve



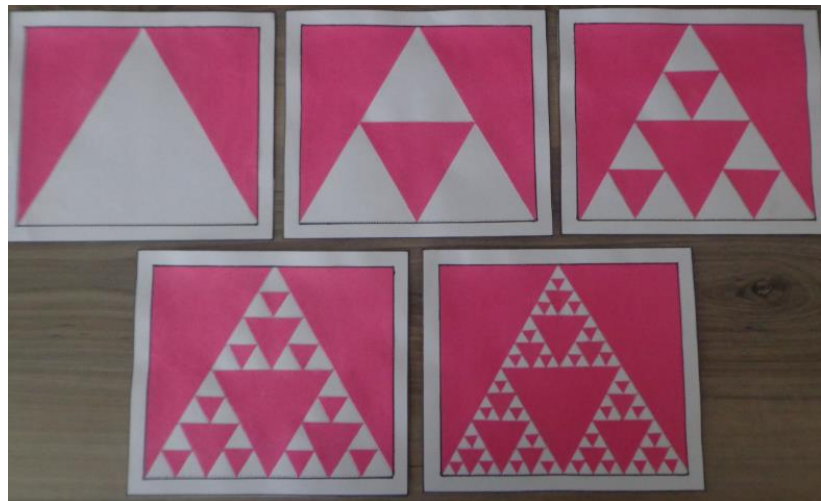
Source: Researchers' file (2019).

Figure 3 – Level 3 of the Koch Curve



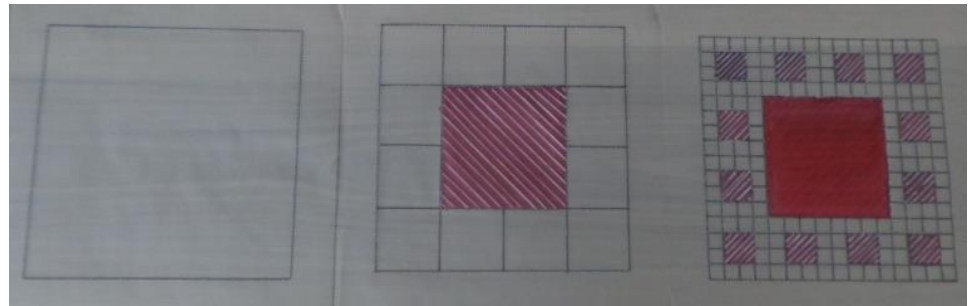
Source: Researchers' file (2019).

Figure 4 – Sierpinski Triangle



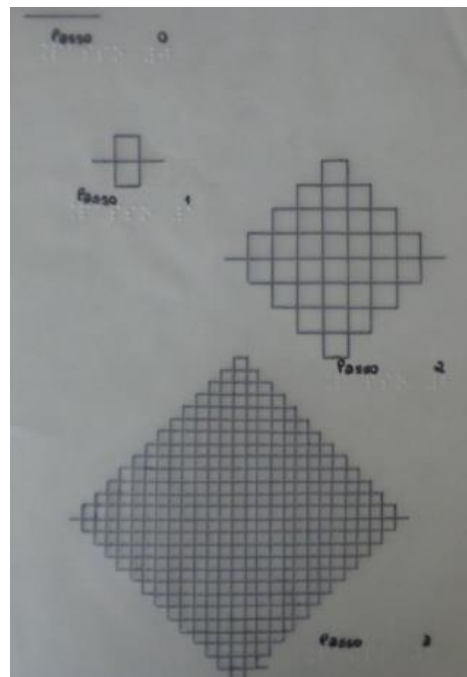
Source: Researchers' file (2019).

Figure 5 – Sierpinski Triangle



Source: Researchers' file (2019).

Figure 6 – Peano Curve



Source: Researchers' file (2019).

RESULTS AND DISCUSSION

According to Vygotski (1998), learning is a social activity in which the student appropriates the knowledge developed historically and culturally, to occur through action and the use of mediating instruments. Hence the need for students to participate interactively in the teaching and learning process, acting on the object of knowledge with the teacher and colleagues. However, many students, with or without disabilities, are not yet able to participate effectively. This is confirmed in the testimonies of the students who participated in this research. According to them, in Mathematics classes, the teacher demonstrates the concepts mechanically, followed by repetitive examples and exercises, decontextualized from life reality. The students Carlos and João complemented by stating that, in many teaching situations, they were excluded of the process, as if they “did not exist” in the classroom.

In this case of study, also extending to all other content taught at school, Fainguelernt (1999) considers that the teaching of geometry through automatism, memorization and repetition does not contribute to the development of geometric thinking, which is necessary for the interpretation of the world in which the human being is inserted. For visual-impaired students, geometric concepts, in addition to contributing to the development of geometric thinking, are fundamental for the understanding of space, location and independent locomotion.

Based on the testimony given by the students surveyed, it was found that they had not appropriated geometric concepts such as regular polygons, lines, segments of lines, parallel lines and others necessary for the study of Fractal Geometry. They recognized, for example, some basic geometric shapes, such as the square, the rectangle, the triangle and the circle, however, they could not conceptualize the figures. For everyone, the square had four equal sides; the rectangle had two equal sides and two different sides, and the triangle had three sides. When presented to them, in tactile materials, different triangles, right triangles, equilateral, scalene and isosceles, they conceptualized only as triangles, not identifying other characteristics such as the relationships between the sides and the internal angles.

In this description, Carlos, when asked what a straight line was, replied: "The one that is not a curve!", demonstrating that he understood that the two geometric elements had differences between them, however, without being able to define them. This data constitutes a zone of proximal development - ZPD, which, according to Vygotski's (1998) assumptions, is the interval between proximal knowledge, which has not yet been internalized by the student, and real knowledge, what the student already has is aware of. Mediation is a way of appropriating knowledge, which the result is development.

These results were also found in other studies, both in visually impaired and non-impaired populations (VIGINHESKI; SILVA; SHIMAZAKI; PACHECO, 2017; VIGINHESKI; SILVA; SHIMAZAKI; ANJOS, 2016; Villa Rouco, Flores, 2013; Nascimento, 2012b). For Nascimento (2012b), citing Pavanello (1993) and Lorenzato (1995), the emptying of geometric contents due to the valorization of algebraic concepts resulting from the Modern Mathematics Movement, as well as the training of teachers, the current proposed curriculum and the influence of textbooks at school are some factors that contribute to the finding described.

The students' answers indicate that they presented some concepts about Geometry, however, they were not enough to mathematically define the geometric shapes. Thus, it appears that, for some reason, there was no appropriation of school concepts by these subjects. It is possible that, in addition to the factors pointed out by Nascimento (2012b), teaching, based only on oral explanations, without interactions between teacher and visually-impaired student, as well as without the use of tactilely adapted materials, has contributed to the fact described. Thus, it can be inferred that teachers of these students did not intervene so that the object of study was consolidated and the result is presented in the gaps in their academic training.

In this study, for the teaching of these concepts, activities were developed such as classification of geometric shapes, identification of the characteristics of each one of them in different objects. We proceeded in the same way for the teaching of other geometric concepts, such as, for example, line, line segment,

parallel lines and perpendicular lines. The students had the concept formed that parallel lines do not intersect and that perpendicular lines do intersect. In this study, such concepts were approached in an articulated way with the knowledge used in daily orientation and mobility, such as parallel and perpendicular streets, among others, so that, in the sequence, the concepts of Fractal Geometry were approached.

When presenting the theoretical aspects of the content discussed, some fractals of nature were used, so that the students could explore with touching, experiencing the perception of the regularities present in them.

In discussions with students about this type of fractal, Bruno explained that when he was able to see, he could not compare trees with any defined geometric shape, because each species has a specific set of characteristics. However, in the process of visual loss, his vision was reduced to seeing figures, and so he began to perceive the treetops as a “ball”, a “circle”. This proves that the lack of vision, even of those who were seers, makes the concept formation process difficult. The mediating instruments and signs help to compensate for some flaws, however, the descriptions and use of mediators are not always enough. In Bruno's case, his lack of vision gave him the idea of a fractal.

The students were asked what infinity was, with a consensus that infinity is what never ends. For students who lost their sight in the course of their lives, it was asked if they could perceive the infinite in phenomena of nature, such as lightning propagated on a stormy day, to which they answered in the affirmative, because its ramifications they seemed to have no end in heaven. It was then asked if the infinite could be perceived through a tactile representation, to which Carlos replied: “I cannot imagine, you can see that it ends”.

Based on Carlos' answer, they were given to the students to explore the elements of nature brought to the class. João, in the same way that his colleague, when exploring objects through touch, realized what was real and what was possible to be explored with his hands. Bruno and Luís were able to imagine the continuity of broccoli and fern, extrapolating the limits of tactile perception. They said they could do it because in the past they were able to see.

In order for this perception to be possible for all participants in the group, a branch of the fern branch was detached, then a smaller fragment, as illustrated in Figure 7:

Figure 7 – Fragmentation of the fern



Source: Researchers' file (2019).

From this fragmentation, Carlos and João could identify the regularity between the whole and the parts, as well as understand the unlimited number of iterations that happen from the first three. From this fact, it was found that the common characteristics of fractals perceived by sight are not so easily perceived by touch. In this way, there is a need to seek other ways of teaching, since nothing was found in other researches.

The concepts about geometric fractals were approached from the Sierpinski triangle - Figure 4. The students, with the exception of Carlos, were able to identify, through touch, the five levels of iterations that were adapted, as described in Figure 4. Carlos was able to identify and recognize a three-dimensional triangle, but not its two-dimensional representation. For Villarouco and Flores (2013), the understanding of representation by the visually impaired person ensures their insertion in everyday life, their mobility and the development of other skills. This statement is valid both for the reading of tactile images and for the mental representation through the abstraction of information obtained.

Carlos, showing concern, declared: "I never dealt with these things at school. [...] I never saw it. For me it's all new. I never saw it. At school they just talked". He added: "The Physics teacher took me to the Physics laboratory, but it was no use. He took it for a take, it didn't show anything." In turn, João was able to identify the tactile adaptations proposed, however, he commented that his ability was related to the fact that, as a child, he developed many tactile stimulation activities with teachers who worked in special education. This finding shows the importance of the teacher in the process of including people with disabilities in regular education, not as responsible for teaching concepts, but in the orientation towards inclusion, to seek together, special education and regular education, to seek solutions so that students with disabilities have their guaranteed right of access, appropriation of knowledge and progression in their school life.

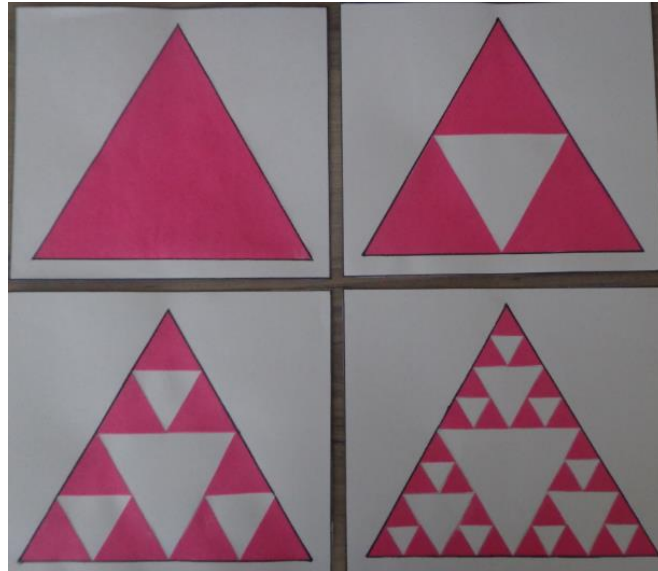
In regular school, like Carlos, João spent a lot of time just listening to the teachers, as he reported. For Liberto, Ribeiro and Simões (2017) and Valente (2008), the ability to recognize tactile drawings by the blind person is related to lived experiences. Liberto, Ribeiro and Simões (2017) also highlight the importance of teaching blind children through the exploration of objects in nature and their surroundings, from the simplest and smallest shapes to complex and larger shapes. From the moment they recognize these objects in three-dimensional format, they start to present their representations through two-dimensional objects. For Fernandes and Healy (2007), geometric knowledge by this portion of students depends, in part, on tactile contact with objects.

It is considered, from the testimonies reported, that the study participants, in a way, were excluded from the learning process when attending regular school, confirming the previous discussion about the need for the teacher to provide the participation of visually-impaired students in the learning process, with the necessary adaptations. João, realizing the difficulties faced by himself and Carlos, commented: *If we didn't learn it in high school, let's do it now.*

It was understood that the difficulty presented by Carlos to identify the different levels of iteration in the Sierpinski triangle was related to the fact that he did not have access to the style of material described here in his schooling process.

Faced with this difficulty, there was a need for a new adaptation to the Sierpinski triangle, so that Carlos could understand the image, as guided by Valente (2008), shown in Figure 8:

Figure 8 – New adaptation of the Sierpinski Triangle



Source: Researchers' file (2019).

From Figure 8, Carlos was able to identify the triangle at level zero and perceive through touching the triangles that were “removed” at each iteration. Thus, he learned that each triangle was taken from the midpoint of each of the sides of the previous triangle, and at each iteration, the number n of triangles taken out was represented by the equivalent power 3^n . The same procedure had already happened with the other students in the previous adaptation, represented by Figure 4. The importance of the teacher to carry out other adaptations is highlighted, as he understands that the one carried out did not reach the proposed objectives, not leaving the visually-impaired student to margin of the learning process. For Fernandes and Healy (2010, p. 1117), “[...] the practices of learners in any learning situation are closely linked to mediating systems, tools, materials and language.”

Another form of adaptation used for visually-impaired students was the description of the image. This form of adaptation is used in entrance exams and contests in general, as well as in other assessments such as the National High School Exam - ENEM, the Brazilian Mathematics Olympiad - OBMEP. In the description, the images are replaced by descriptive texts, in the written and oral modalities. Here, we resort to the descriptive adaptation of two fractals, the Sierpinski Triangle and the Koch Curve, as explanatory samples of the processes. For the first, the description is used after the development of activities with the adapted materials in relief. For the second, the description is used in the introduction of the concept, before the use of tactile adaptations. Fractals are described through the texts presented:

Sierpinski Triangle

Level 0: An equilateral triangle whose sides measure 14 cm.

Level 1: The equilateral triangle was divided into four equilateral triangles from the midpoint of each of the sides that formed the triangle at level 0. From these four triangles, the central triangle was removed.

Level 2: The process performed in Level 1 is repeated for each of the three equilateral triangles; that is, each of them was divided into four equilateral triangles from the midpoint of their sides, and the central triangle removed.

Level 3: The process was repeated for each of the remaining triangles.

Repeating this process over and over again characterizes the Sierpinski Triangle as a fractal.

Curva de Koch

Level 0: A line segment AB.

Level 1: The line segment AB was divided into three equal parts, with the same measure. In the central part, an equilateral triangle was built and, later, its base was eliminated.

Level 2: In each of the four segments resulting from Figure 2, the same procedure is performed. Divide each one into three equal parts. In the central part, an equilateral triangle is constructed, eliminating its base.

Level 3: All segments are divided into three parts again, and equilateral triangles are constructed in the central part of each segment. The base of each of the triangles is eliminated.

The use of the description of the Sierpinski Triangle fractal contributed to the consolidation of the concepts discussed, as the students commented that from this experience it was possible to make a mental image of the images they had explored through touch. João commented that he was able to understand the iterations infinite times: "Wow, but this triangle is going to end as it is going to be divided", that is, he was able to understand that the more iterations there were in the image, the smaller the triangles would be, until they were not perceptible to the touch.

Regarding the Koch Curve, because the description was used in the first moment, the students said that they could not imagine the figures, with the exception of the image presented at Level 0, a straight line segment. Miguel traced the image with his finger on the table, while the text was read. However, the design traced by his finger was a continuous line, on which the student drew the triangles. It is possible that he managed to make a sketch based on the visual experiences he had, still retained in his memory, but the description presented alone was not enough for him to sketch the image as it really was.

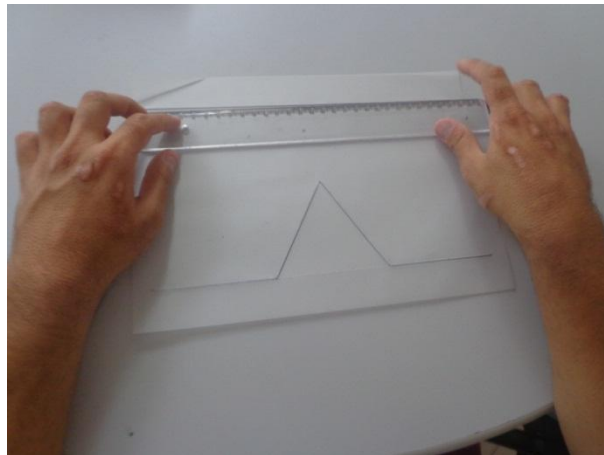
The lack of access to the tactile image before the description is not solely responsible for the fact that students are not able to make a corresponding mental image. Valente (2008), when presenting to blind students a pictogram of a ladder adapted in a tactile way, found that most of them recognized the image as the representation of the object, as a function of motor memory. For Damásio (1999), cited by Valente (2008), mental images can also be formed by elements from all

the senses involved in the interaction between the person and the environment, not only through vision.

When the students in this study had access to the tactile images, they commented on the difficulty in imagining something when they did not have access to the tactile material beforehand. For the subjects, the difficulty is present when they take part in contests, such as entrance exams, whose tests, most of the time, present only the description of the images through text, unaccompanied by images in relief. According to them, it was much easier to understand the description of the Sierpinski triangle with the description, since they had already had a tactile experience about themselves.

Also noteworthy is the fact that students Carlos and João did not know how to use measuring instruments, such as a ruler, for example. The other three students, having been seers for part of the school period, knew how to use them. Some activities developed with the Koch Curve and Sierpinski Carpet fractals asked the students to the measurements, for this, the adapted ruler in relief was used, so that the blind students could perform the task, and, from the collection of measurements, Calculate the perimeter and area of the figures. The adaptations made to the materials allowed such action procedures. Figure 9 shows the image of a student taking measurements with the adapted ruler:

Figure 9 – Measurement of the Koch Curve's segments



Source: Researchers' file (2019).

The results point to the appropriation of the knowledge covered in this study through the use of tactile tools, highlighting the importance of its use for the teaching of other contents. Nascimento (2012b) also noted the importance of using representation and the use of manipulative materials for the introduction of concepts with the non-visually-impaired students, in line with what Galperin (2009) proposes, about a way of teaching that contemplates the action on the objects or their representation as a first step towards the formation of action on the mental plane, the internalization of concepts, to guarantee the appropriation of knowledge. For Fernandes and Healy (2009), tactile tools not only facilitate mental processes, they fundamentally shape and transform such processes.

The need for specific training for Mathematics teachers is also underlined, for the inclusion of students with disabilities, both in the initial period and throughout their career, on a continuous basis. Fernandes and Healy (2010) warn that many

teachers claim to be unprepared for inclusion. In fact, there are gaps in their training and the result is the exclusion of students in the regular classroom, even unconsciously by the teacher.

Although research in the area has increased quantitatively and qualitatively, it is necessary to intensify training policies by the competent departments. Universities are responsible for developing research and extension projects, research, and study groups. Among others, actions that promote training for teachers on working with visual impairments and the necessary adaptations for their teaching, in addition, of course, to access to research, as they allow them to be instrumentalized for inclusive teaching, such as, for example, the research¹ that relates to the theme described in this study, which seeks in an equitable way tangible initiatives for the transformation of the process of education and development of teachers.

FINAL CONSIDERATIONS

This research happened in a Special Education school, out of the context of regular school inclusion. It is important that it is also conducted in regular education classes, in which the students investigated here are included, in order to validate the adaptations presented, since it is in this social context that students have access to knowledge in an official way.

The materials that were developed for the research are simple examples of possibilities for pedagogical work and do not require greater financial investments, to be made by the teachers themselves.

Dessa forma, os docentes promovem um ensino inclusivo na área de Matemática, não só para os estudantes com deficiência visual, mas para todos os estudantes de forma ampla.

In addition, other studies can be performed making use of adaptations for other mathematical contents, to equip regular education teachers to deal with mathematical concepts in a situation of inclusion. This instrumentalization can happen in order to encourage teachers to develop new tools for teaching blind people, and not only, presenting them with what already exists. This is in line with the proposal of the project “Rethinking Teacher Education: Foresting Inclusive Practices for Visually-Impaired Students in Mathematics Classes”, with the aim of understanding what happens when forms (e.g. teaching materials and practices) of teaching mathematics that support the development and learning of visually-impaired are conceived, developed and implemented in collaboration between teachers and researchers, bringing together practice and research purposefully and systematically, providing transformations in the process of teacher education.

Finally, this study contributes to the teaching and learning process of visually impaired students who attend regular education, as it provides access to mathematical concepts and their participation in the process of forming formal knowledge. Likewise, it equips Mathematics teachers for the inclusive teaching of the discipline to contribute to the areas of Inclusive Education, Special Education and Mathematics Education.

ENSINO DE CONCEITOS SOBRE GEOMETRIA FRACTAL PARA ESTUDANTES CEGOS: DO ESTUDO DE CASO À INSTRUMENTALIZAÇÃO DOCENTE

RESUMO

A pesquisa de intervenção aqui relatada, de natureza aplicada e abordagem qualitativa, tem como objetivo analisar as contribuições das adaptações materiais, táteis e descritivas, no processo de ensino e aprendizagem de Geometria Fractal para estudantes com deficiência visual, a fim de possibilitar-lhes acesso ao conteúdo escolar sobre esse tema. Esses sujeitos necessitam de ferramentas adequadas que lhes permitam o acesso e a apropriação dos conhecimentos científicos matemáticos. Dessa forma, desenvolveu-se um estudo prático, em um centro de atendimento educacional especializado na área da deficiência visual, em um município do Centro-Oeste do Paraná, com um grupo de cinco pessoas cegas e com baixa visão, já concluintes do Ensino Médio. A coleta de dados da intervenção pedagógica deu-se por meio de observações, registros em filmes e diário de campo, examinados por meio da análise da conversação. Os resultados indicaram lacunas na formação escolar dos estudantes em relação aos conceitos matemáticos necessários para a aprendizagem de Geometria Fractal. O estudo constatou que a utilização de materiais adaptados para o ensino de conceitos relacionados a esse conteúdo, como as imagens táteis contribuíram para a formação dos conceitos ensinados. Os resultados apontam a necessidade de promover formação para os professores de Matemática na área de Educação Inclusiva, com vistas à efetivação da inclusão e à apropriação dos conhecimentos científicos pelos estudantes com tal deficiência. Dessa forma, os resultados obtidos neste estudo corroboram com o desenvolvimento do projeto Rethinking Teacher Education: Foresting Inclusive Practices for Visually-Impaired Students in Mathematics Classes, fomentado pela Universidade de Columbia e Fundação Lemann.

PALAVRAS-CHAVE: Ensino de Matemática. Deficiência visual. Adaptações táteis. Geometria Fractal.

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NOTES

1 Rethinking Teacher Education: Foresting Inclusive Practices for Visually-Impaired Students in Mathematics Classes.

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Mailing address: Lucia Virginia Mamcasz Viginheski - lmamcaszviginheski@gmail.com

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