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Instructional Design in the development of a Digital Didactic Sequence on the topic of Derivatives theme

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ABSTRACT

This study investigates the potential of developing a Digital Didactic Sequence (DDS) with different technological resources, applied to students in the field of Exact Sciences who attended the Differential and Integral Calculus I discipline. The general objective of the research was to analyze the contributions of a DDS, with the theme of Derivatives, aiming to identify the difficulties of the students and to broaden the understanding of the concepts and their application in problem-solving situations for students in the field of Exact Sciences. The methodology applied is qualitative, with the development of an experiment involving 7 higher education students. The DDDs were developed on websites and made available on the Siena System. The activities consisted of study material in PDF format, an explanatory video of the study materials, complementary videos available on YouTube and, in some concepts, educational objects available in the GeoGebra software. The DDDs were based on Contextualized Instructional Design. Based on the data analysis, from the Siena system database, it was possible to identify the concept in which the students presented the greatest difficulty - Derivatives - Chain rule. In the concepts of Basic Mathematics – Functions, Direct Derivatives, Product and Quotient, Applications of Derivatives in Solving Problem Situations, different students studied the Digital Didactic Sequences to review the concepts in which they did not achieve a satisfactory performance. The concepts of Basic Mathematics – Arithmetic and Basic Mathematics – Algebra were not used during the research because the seven students achieved an expected performance in the first adaptive test performed. The research participants considered that the interface was intuitive, with materials and resources considered suitable forstudying.

KEYWORDS: Derivatives. Instructional Design. Digital Didactic Sequence.

INTRODUCTION

Discussion about the degree of difficulties that Higher Education students have in relation to the mathematical contents of Basic Education is a subject that permeates the academy, seeking adequacy for the development of competences and which methodologies are considered important to reduce such difficulties and advance in mathematical concepts in higher education. According to Masola and Allevato (2016), access to higher education institutions was democratized and many students entered university classrooms, arriving with different goals and abilities, clearly showing deficiencies in training and/or mastery in learning mathematical concepts.

Therefore, it becomes important to reflect on teaching strategies and resources that enable the development of teaching and learning of mathematical concepts in Higher Education. This article addresses the Derivatives theme, seeking methodological alternatives for the student to build knowledge by applying them in problem solving, not prioritizing the memorization of formulas and algorithms for solving activities that are reduced to memory exercises with no practical application, also seeking to reduce the difficulties related to the previous mathematical concepts necessary for the study of this theme.

It is understood that the use of Digital Technologies (DT), through Hybrid Teaching, can be a path that enables students to develop according to their routine and study pace, allowing them to revisit and/or study concepts at different times and according to their learning pace. In this way, it is possible to advance according to their personal performance.

This article presents the design of Digital Didactic Sequences (DDS) developed on Google Sites, structured according to the principles of Contextualized Instructional Design (CID) (FILATRO, 2008), which adapt to different electronic devices (laptops, tablets, smartphones, etc.). The research question was: How to develop a DDS, with the theme Derivatives, aiming to identify the difficulties and enhance the understanding of concepts and their application in problem-solving situations for students who have already taken the Differential and Integral Calculus I course in the area of Exact Sciences?

THE TEACHING OF CALCULUS AND MATHEMATICS EDUCATION IN HIGHER EDUCATION

Approaching Mathematics Education in Higher Education, researchers such as Homa (2019), Igliori and Almeida (2015), Cabral and Baldino (2006), brought discussions related to the teaching of the Derivatives theme in Differential and Integral Calculus I classes, of courses in higher education graduation. The debate on the teaching and learning of this theme is quite complex, as there are, on the part of teachers, expectations regarding students' prior knowledge that, in general, are not met (LOPES; REIS, 2019; MASOLA; ALLEVATO, 2016; CURY; CASSOL, 2004).

The lack of prior knowledge on the part of the students, classrooms with many students, excess of contents to be worked on and, often with methodologies that favor procedures with explanation-exercises, has meant that the rate of failure and dropout, in the disciplines of Differential and Integral Calculus, for many years, remain high (BRAGG, 2005; REIS, 2001).

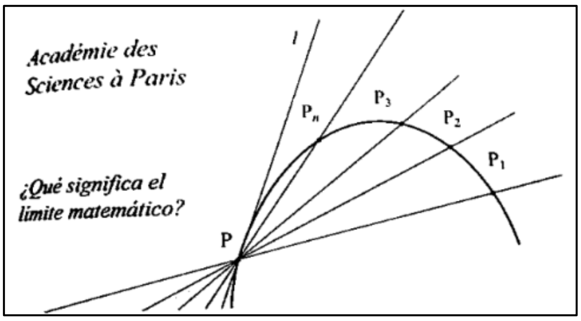
According to the authors Baldino and Fracalossi (2012), Reis (2009), Cabral and Baldino (2006), Reis (2001) many Calculus concepts are worked on in the classroom based on a formalism imposed by the presentation of contents, addressing the explanation-exercise method, through definitions, theorems, demonstrations and properties, and then the student must solve exercises as a way of memorizing the procedures, which leads to a fragmented understanding, in a way that the student often does not can apply the concepts in solving problem situations.

Reis (2001) points to epistemological problems associated with learning Calculus, as well as problems of a didactic nature, excessive formalism, lack of didactic transposition, etc., not contemplating the application of concepts, which makes the study of little significance for the student (HOMA, 2019; CANTORAL, 2013).

Lopes and Reis (2019) highlight that often students perform algebraic operations and memorize formulas to derive a function, but do not understand the mathematical object being studied. Cantoral (2013) addresses that this type of error is made by students because, generally, the presentation of Derivatives content is done through the formal definition of the derivative with the calculation of the limit and an example of a four-step rule to teach the techniques of derivation. The four-step rule is an operative way in which students are taught the method for obtaining derivation techniques: First step: calculate $f(x+h)$, to h an increment in x; Second step: determine the increment of the function, calculating $f(x+h)-f(x)$; Third step: calculate the mean change by the increment ratio $\frac{f(x+h)-f(x)}{h}$; Fourth step: Calculate the variation for the increment h tending to zero $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

This is the rule, traditionally used in the educational environment and, graphically (Figure 1), it is presented following the model developed by D'Alambert in 1748.

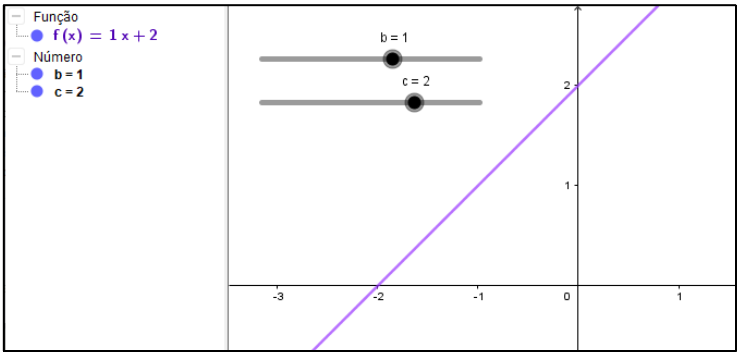
Figure 1 - Example of the limit given by D'Alambert in 1748



Source: Cantoral (2013, p. 193).

Cantoral (2013) proposes to start the study of tangent lines using technological resources, in which students use software to draw a line (which will be the tangent) at a given point, as shown in Figure 2.

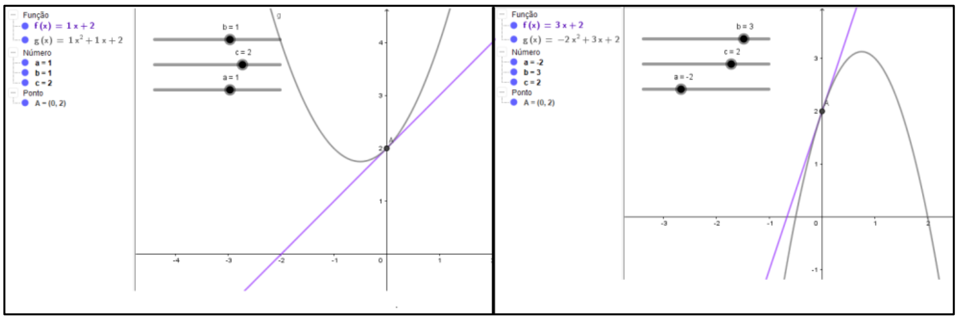
Figure 2 - Line $y = x + 2$, tangent to y at $(0, 2)$



Source: Santos (2021, p. 50).

From this tangent line to the y-axis at the point (0,2), students must find equations of parabolas that are tangent to this line at this same point. In Figure 3, examples are presented with parabolas that are tangent to the line $y = x + 2$ at the point $(0, 2)$. Cantoral (2013) stresses that students must autonomously manipulate the coefficients of the parabolas so that they can understand their variations, and in a group work, proposing that the variation of the coefficients, allows students to observe the point of tangency between the parabola and the y axis will also change, which will give rise to a different equation of the first degree, of the type $y = x + 2$.

Figure 3 – Examples of tangent parabolas to the line $y = x + 2$ at the point $(0, 2)$



Source: Santos (2021, p. 50).

According to Cantoral (2013) this activity allows students to build an initial idea of tangency between the curve (parabola) and the straight line, but without entering the mathematical definitions of this sense. Thus, students are expected to observe that the linear term of the parabola will be the formula of the tangent line to the origin curve, that is, $ax^2 + bx + c \leftrightarrow bx + c$.

In the Design of the DDS developed, we sought to use the concepts and geometric interpretation as a way of understanding, and then use the algebraic resolution of the problem situations, using digital resources that would allow the student to carry out the aforementioned visualization.

INSTRUCTIONAL DESIGN

Instructional Design (ID) is understood as teaching and learning planning which includes activities, strategies, assessment systems, methods and instructional materials (FILATRO, 2008). ID is an area of activity that is directly linked to Education, more precisely, to the production of didactic materials, it can be described as a methodology that emerged from the new practices of pedagogical practice and since then place the student at the center of the teaching and learning process.

Filatro (2008) presents the term Instructional Design, explaining the terms individually. Design becomes a product (the result of a process or activity), and instruction is the teaching activity that uses communication to facilitate learning. The author defends ID as the intentional and systematic action of teaching that involves the planning, development and application of methods, techniques, activities, materials, events and educational products in specific didactic situations, in order to promote, based on the principles of learning known to human learning. It is the process of identifying a learning problem and designing, implementing and evaluating a solution to it.

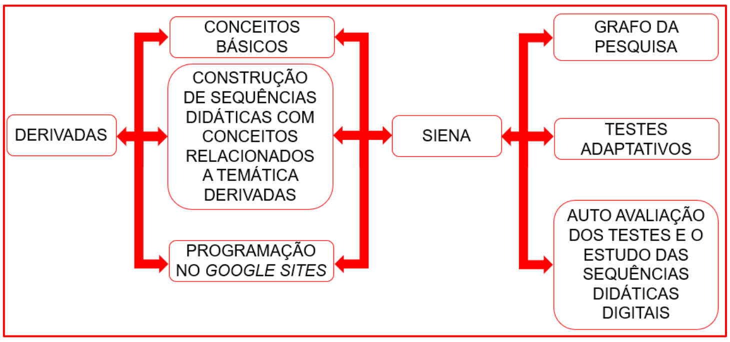
It is understood that ID can be defined as the set of activities involved in the formulation of an educational action. Thus, it is not a single task, but a diversity of practices that allow the construction of a qualified educational product, which meets not only the specificities of the students, but also the pedagogical orientation of the educational institution (FILATRO, 2008). Groenwald (2020) argues that, supported by technologies, ID supports effective contextualization mechanisms, characterized by: greater personalization to individual learning styles and rhythms; adaptation to institutional and regional characteristics; updating based on constant feedback; access to information and experiences external to the teaching organization; possibility of communication between process agents (teachers, students, technical and pedagogical staff, community); and automatic monitoring of the individual and collective construction of knowledge, following the indications of Filatro (2008).

In this research, Contextualized Instructional Design (CID) was chosen, which seeks balance and automation of planning processes, personalization and contextualization in the developed didactic situation. The term CID was used to describe the intentional action of planning, developing and applying specific didactic situations that, taking advantage of the potential of a Virtual Learning Environment (VLE), incorporate, both in the design phase and during implementation, mechanisms that favor contextualization and flexibility, making it possible for students to follow different paths, according to their learning pace and according to their learning preferences. When developing the DIC, it was decided to organize a Didactic Sequence (DS), which are sets of activities that are ordered, structured and articulated with each other with the aim of optimizing the teaching and learning process, integrating learning and assessment activities (ZABALA, 1998).

Dolz and Schneuwly (2004) classify a DS as a group of activities designed and organized by the teacher, aiming to achieve a learning objective, in which the particular order of these activities is essential for the learning process since the final result does not depend on the content of each task, but how all of them are organized in the DS, allowing the student to go through their learning paths and advance in understanding what is proposed in the activities. For Groenwald, Zoch and Homa (2009), an DDS enables the use of different resources, with a higher quality standard such as video examples, texts and animations, that is, a higher quality visual content. When working in this way, students no longer receive the same content simultaneously and start to follow different paths, according to their student profile and their performance. Figure 4 shows the CID model applied in the development of the DDS with the Derivatives theme, developed in the SIENA intelligent system.

The SIENA system is an intelligent system that is able to communicate information about the students' knowledge regarding the research theme, and aims to assist in the process of recovering mathematical content, using the combination of conceptual maps and adaptive tests (GROENWALD; RUIZ, 2006). For the authors, this system allows the teacher to analyze the level of prior knowledge of each student, enabling teaching planning in accordance with the reality of the students, which can provide meaningful learning. The computer process makes it possible to generate an individualized map of the students' difficulties, which will be linked to hypertexts, which will be presented when the student has difficulties, allowing them to recover from such difficulties.

Figure 4 – Contextualization movements in the Instructional Design proposal



Source: Santos (2021, p.59).

The DDS were implemented in the SIENA system, where the following actions are necessary: development of a graph with the concepts to be developed; question bank for the Adaptive Tests for each graph concept; Didactic sequences for each concept of the graph. A computerized adaptive test is administered by the computer, which seeks to adjust the test questions to the skill level of each examinee. According to Costa (2009), a computerized adaptive test seeks to find an optimal test for each student. The purpose of the adaptive test is to manage questions from a bank of previously calibrated questions, which correspond to the examinee's ability level.

When working in a computerized learning environment, students are allowed to develop their activities according to their learning pace, in which contents will be presented with different methodological resources, important for the development of mathematical skills and, according to their choices and preferences, respecting individual performances (GROENWALD; ZOCH; HOMA, 2009). In this way, the teacher has an important role, which is to organize the planning of activities that make up the DDS, linking and articulating the different activities throughout a didactic unit, allowing the student to make their choices throughout the study (ZABALA; ARNAU, 2010). The term DDS is used as a set of activities organized and developed using digital resources. The ID developed with the research theme is presented below.

ID AND THE VIRTUAL LEARNING ENVIRONMENT WITH DERIVATIVE THEMES

The DDSs are indicated for higher education students, aiming to study the concepts related to the Derivatives theme and allowing students to revisit the previous concepts necessary for learning the Derivatives theme. DDS were developed for each concept of the Research Graph, continuing the research by Silva (2019), where the graph and the question bank for the adaptive tests were developed.

The Graph has six concepts: Basic Mathematics – Arithmetic; Basic Mathematics – Algebra; Basic Mathematics – Functions; Direct Derivatives, Product and Quotient; Derivative - Chair rule; Derivative Applications with Resolution of Problem Situations.

The Adaptive Tests are organized into six (6) question banks, consisting of sixty (60) questions in each concept of the graph, multiple choice, classified into three levels of difficulty (easy, medium and difficult), with 20 questions for each difficulty level. To classify the questions, Silva (2019) used the categorization presented in Chart 1.

Chart 1– Categorization of questions by concepts

Concepts	Levels	Characteristics of the questions
Basic Mathematics: Arithmetic, Algebra and Functions	Easy	They only involve a concept or a procedure for its resolution..
	Medium	They only involve a concept or a procedure for its resolution.
	Difficult	Three or more procedures and strategies are needed to perform the questions, in addition to requiring a higher level of abstraction from the students.
Direct Derivatives, Product and Quotient Rules	Easy	Direct derivatives of the form and with simpler functions.
	Medium	Derivatives considered simple with application of the product and quotient rules, involving algebraic functions with power and root, trigonometric, exponential and logarithmic.
	Difficult	Derivatives of complex functions that are solved by applying the product and quotient rules.
Derivatives – Chain Rule	Easy	Derivatives of simple functions solved using the chain rule.
	Medium	Derivatives of complex and composite functions, solved using the chain rule.
	Difficult	Derivatives of more complex, difficult and composite functions, solved by applying the chain rule.
Application of Derivatives with Resolution of Problem Situations	Easy	Concepts to interpret in each problem situation, resolution of direct derivatives of the form.
	Medium	Interpretation of problem situations, resolution of derivatives, followed by specific calculations of the function, replacing values found by performing the necessary application.
	Difficult	Derivation resolution of the most complex functions and analysis of maximum and minimum points, applied in problem situations.

Source: Silva (2019, p. 74).

Chart 2 presents examples of questions with the concepts of Application of Derivatives with the concept of Resolution of Problem Situations.

Chart 2- Example of Derivatives questions with Problem Solving

FÁCIL
7. A city X is hit by an epidemic disease. The health sectors calculate that the number of people affected by the disease after a time t (measured in days from the first day of the epidemic) is approximately given by $f(t) = 64t - \frac{t^3}{3}$. What is the rate of expansion of the epidemic disease in 4 days?
MEDIUM
13. A water reservoir is being emptied for cleaning. The amount of water in the reservoir, in liters, t hours after the flow has started is given by $V = 50(80-t)^2$. Determine the rate of change of the volume of water in the reservoir during the first 10 hours of runoff.
DIFFICULT
8. A farmer wants to build a shed on his farm. Doing the calculations, he comes to the conclusion that the cost of the work in reais is given by the function $C(x) = 10x + \frac{160}{x}$, where x is the measure in meters of the side of the shed. What should be the length x of the side of the shed to keep the cost to a minimum? What will this minimum cost be?
EASY
7. Uma cidade X é atingida em uma moléstia epidêmica. Os setores de saúde calculam que o número de pessoas atingidas pela moléstia depois de um tempo t (medido em dias a partir do primeiro dia de epidemia) é, aproximadamente, dado por $f(t) = 64t - \frac{t^3}{3}$. Qual a taxa de expansão da moléstia epidêmica em 4 dias?
MÉDIO
13. Um reservatório de água está sendo esvaziado para limpeza. A quantidade de água no reservatório em litros, t horas após o escoamento ter começado é dado por $V = 50(80-t)^2$. Determine a taxa de variação do volume de água no reservatório durante as 10 primeiras horas de escoamento.
DIFÍCIL
8. Um fazendeiro quer construir um galpão em sua fazenda. Fazendo os cálculos ele chega à conclusão que o custo da obra em reais, é dado pela função $C(x) = 10x + \frac{160}{x}$, onde x é a medida em metros do lado do galpão. Qual deverá ser a medida x do lado do galpão para que o custo seja mínimo? Qual será esse custo mínimo?

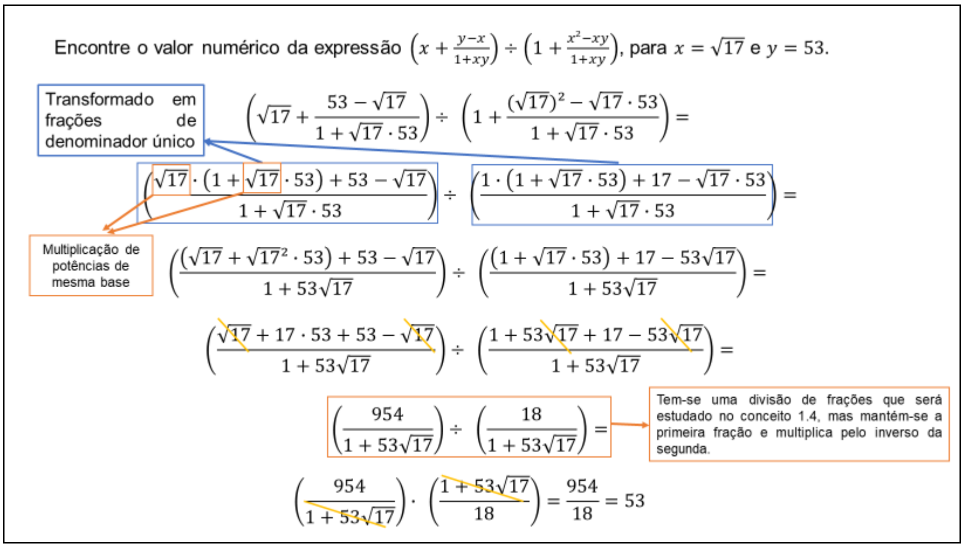
Source: Silva (2019, p. 188-199).

The SDD, implemented in the SIENA system, were developed on Google Sites, using the resources of: PowerPoint material with explanatory video allowing the student to have the same material in written and video format, thus being able to choose the way he prefers; indication of complementary videos of studies; learning objects in the GeoGebra software (in the concepts System of Equations, Matrices, Affine Function, Quadratic Function, Exponential Function and Logarithmic Function).

In the available study materials, the activities were organized in order to complement each other, aiming to enable the student to study the concept presented and, also, the possibility of studying the previous concepts necessary to understand the concept. For example, for a difficult level question, in which it is requested to find the function derived from $y = \sqrt{e^{2x} + 2x}$, the student must have an understanding of the composition of the functions $f(x) = \sqrt{x}$ and $g(x) = e^{2x} + 2x$, that is $y = f(g(x))$.

To differentiate the function $f(x)$, the student must be able to work with the potentiation/radiation properties, so that he can differentiate the function. In this sense, Figure 5 shows an example of the chain of activities.

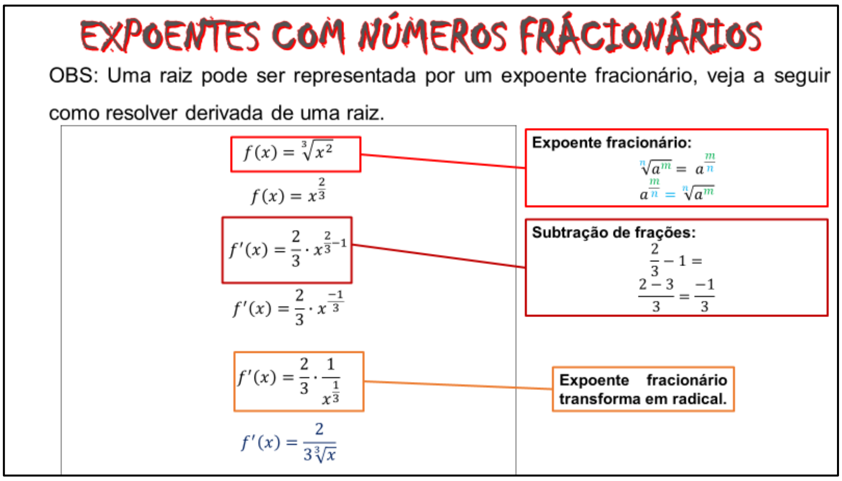
Figure 5 – Using the properties of Potentiation and Rooting operations



Source: Santos (2021, p.69).

So that the student can then derive the function $f(x) = \sqrt{x}$ q which becomes $f(x) = x^{\frac{1}{2}}$, he needs to develop the derivative of Power. And, to help the student, there is the following example in the Didactic Sequence (Figure 6).

Figure 6 – Strategies for Power Derivative content



Source: Santos (2021, p. 111).

That said, the student still needs to work with the function $g(x) = e^{2x} + 2x$, before working with the $y = f(g(x))$. function. To work with the derivative of the following functions, in the material of the Didactic Sequence there is the Derivative of a power and the derivative of the exponential function. Figure 7 shows an example available in the Didactic Sequence, on the derivative of an exponential function. For the Derivative of power, an example has already been presented in the previous figure.

Figure 7 – Strategies for the content of the Derivative of Exponential Functions

DERIVADA DA FUNÇÃO EXPONENCIAL

Tendo uma função:

$$f(x) = e^x$$

A derivada desta função é:

$$f'(x) = e^x$$

As funções, costumeiramente, aparecem com a variável u, veja a situação:

$$f(u) = e^u$$

$f'(u) = e^u \cdot du$, sendo du a derivada de u.

Assim, para $f(x) = e^x$, tomamos $x = u$

$$f(x) = e^u$$

$$f'(x) = e^u \cdot \frac{du}{dx}$$

Se $u = x \Rightarrow \frac{du}{dx} = 1 \Leftrightarrow du = 1dx$

Logo $f'(x) = e^x.1$

$$\boldsymbol{f'(x) = e^x}$$

Source: Santos (2021, p. 114).

Finally, the student must work on the derivative of composite functions and, for this, examples are available as shown in Figure 8.

Figure 8 – Strategies for the content of Derivatives of Composite Functions

5.1 DERIVADA QUE ENVOLVAM REGRA DA CADEIA

Nesta seção você estudará derivadas de funções compostas, utilizando as regras de derivação já trabalhadas.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

DERIVADAS QUE ENVOLVAM REGRA DA CADEIA

Tendo a função $f(x) = \left(x^2 - \frac{1}{x^2}\right)^6$

Considera-se $x^2 - \frac{1}{x^2} = c$

$$f(c) = c^6$$

$$f'(c) = 6 \cdot c^{6-1} \cdot dc$$

Sabendo que $c = x^2 - \frac{1}{x^2} \Rightarrow \frac{dc}{dx} = 2x + \frac{2}{x^3}$

Substituindo a derivada da função:

$$f'(c) = 6 \cdot c^5 \cdot \left(2x + \frac{2}{x^3}\right) \Leftrightarrow Aplicando\ a\ distributiva\!: f'(c) = \left(12x + \frac{12}{x^3}\right) \cdot u^5$$

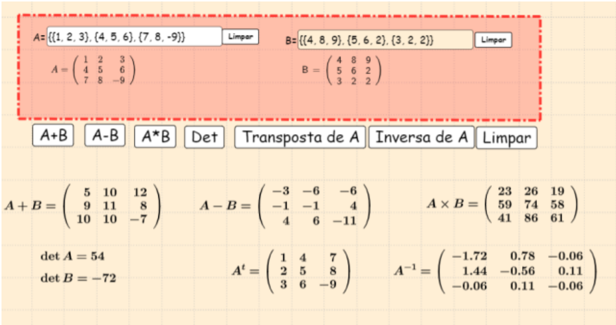
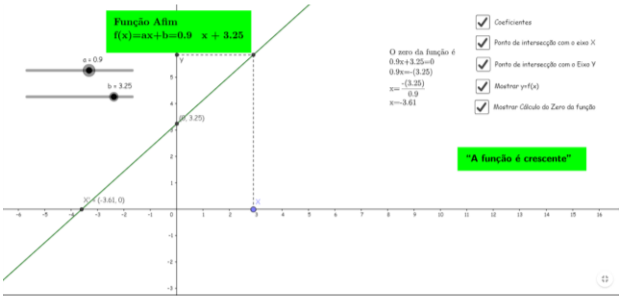
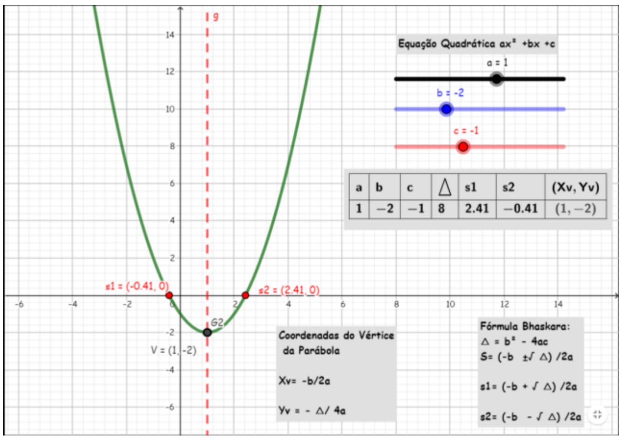
$$\boldsymbol{f'(x) = \left(12x + \frac{12}{x^3}\right) \cdot \left(x^2 - \frac{1}{x^2}\right)^5}$$

Source: Santos (2021, p. 119).

All study materials have an auxiliary video for each content, available on YouTube. The option to present explanatory videos, referring to the study material and complementary videos, followed the ideas of Borba, Domingues and Lacerda (2015) who indicate that videos are present in the students' study routine. Fontes (2019) addresses the issue of how easy it is to see, review, pause, analyze and intervene (pausing, changing the pace or sequence of images).

Figure 9 presents examples of learning objects made available in the CID of the DDS.

Figure 9 - Learning objects available in the CID of the DDS

	In this figure, material available in GeoGebra for the study of operations with matrices (addition, subtraction and multiplication), transposed matrix, inverse and the determinant of a matrix is presented.
	In this figure there is material available in GeoGebra in which students can manipulate the coefficients of the function and, thus, study the point of intersection with the x axis, with the y point, the zero of the function and analyze whether the function is increasing, decreasing or constant.
	In this figure, the student has the possibility of studying the variation of the coefficients of the Quadratic Function, the delta, the roots, the vertex and the interval in which the function is increasing/decreasing.

Source: Research Data (2021).

As in Silva's research (2019), students have difficulties in interpreting problems, which leads to difficulties in their resolution. For this, throughout the DDS, strategies were drawn up with the bias of algebraic manipulation and problem solving, seeking to enable the expansion of understanding of the use of algebraic language. Figure 10 presents an example of a problem about the concept of Derivatives, based on the kinematic interpretation.

Figure 10 – Solving Problems Involving Derivatives and Kinematic Interpretation

INTERPRETAÇÃO CINEMÁTICA DA DERIVADA

Suponha que um objeto se movimentando sobre uma linha reta de acordo com a equação $s = f(t)$, onde $\boldsymbol{s}$ é o deslocamento do objeto a partir da origem do instante $\boldsymbol{t}$.

A função f que descreve o movimento é chamado **função posição** do objeto. No intervalo de tempo entre $t = a$ e $t = a + h$, a variação na posição será de $f(a + h) - f(a)$. A velocidade média nesse intervalo é:

$$velocidade\ média = \frac{deslocamento}{tempo} = \frac{f(a+h) - f(a)}{h}$$

Suponha que a velocidade média seja calculada em intervalos cada vez menores $[a; a + h]$ ou seja, fazendo h tender a zero.

Então, a expressão da inclinação da reta tangente calculada a velocidade instantânea no ponto a é:

$$v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Isso significa que a velocidade no instante $\boldsymbol{t} = \boldsymbol{a}$ é igual à inclinação da reta tangente em $\boldsymbol{P}$ ou seja, é a derivada da função posição nesse instante.

$$s'(a) = v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

EXEMPLO COM INTERPRETAÇÃO CINEMÁTICA

1) Um móvel se movimenta conforme a equação da posição $s(t) = 3t^2 - 8t + 3$, em unidades do Sistema Internacional. Calcule sua velocidade nos instantes $t = 1$ segundo e $t = 3$ segundos.

A derivada da equação da posição resulta na velocidade instantânea, assim basta calcular a derivada nos instantes apresentados na questão.

Quando $t = 1$ segundo: Têm-se $a = 1$.

$s(1) = 3 \cdot 1^2 - 8 \cdot 1 + 3$

$s(1) = 3 - 8 + 3$

$s(1) = -2$ m

$s(1 + h) = 3 \cdot (1 + h)^2 - 8(1 + h) + 3$

$s(1 + h) = 3 \cdot (1 + 2h + h^2) - 8 - 8h + 3$

$s(1 + h) = 3 + 6h + 3h^2 - 5 - 8h$

$s(1 + h) = 3h^2 - 2h - 2$

Substituindo no limite: $s'(1) = v(1) = \lim_{h \to 0} \frac{s(1+h) - s(1)}{h}$

$$s'(1) = v(1) = \lim_{h \to 0} \frac{3h^2 - 2h - 2 - (-2)}{h}$$

$$v(1) = \lim_{h \to 0} \frac{3h^2 - 2h}{h}$$

$$v(1) = \lim_{h \to 0} 3h - 2$$

$$v(1) = 3 \cdot 0 - 2$$

$$v(1) = -2$$

Logo, a velocidade no instante $\boldsymbol{t = 1}$ segundo é $\boldsymbol{v = -2\ m/s}$

Ter uma velocidade negativa representa que o objeto está indo em sentido contrário/inverso ao adotado.

Quando $t = 3$ segundos: Têm-se $a = 3$.

$s(3) = 3 \cdot 3^2 - 8 \cdot 3 + 3$

$s(3) = 27 - 24 + 3$

$s(3) = 6$ m

$s(3 + h) = 3 \cdot (3 + h)^2 - 8(3 + h) + 3$

$s(3 + h) = 3 \cdot (9 + 6h + h^2) - 24 - 8h + 3$

$s(3 + h) = 27 + 18h + 3h^2 - 21 - 8h$

$s(3 + h) = 3h^2 + 10h` + 6$

Substituindo no limite: $s'(3) = v(3) = \lim_{h \to 0} \frac{s(3+h) - s(3)}{h}$

$$s'(3) = v(3) = \lim_{h \to 0} \frac{3h^2 + 10h + 6 - 6}{h}$$

$$v(3) = \lim_{h \to 0} \frac{3h^2 + 10h}{h}$$

$$v(3) = 3h + 10$$

$$v(3) = 3 \cdot 0 + 10$$

$$v(3) = 10$$

Logo, a velocidade no instante $\boldsymbol{t = 3}$ segundo é $\boldsymbol{v = 10 m/s}$

Source: Santos (2021, p. 109).

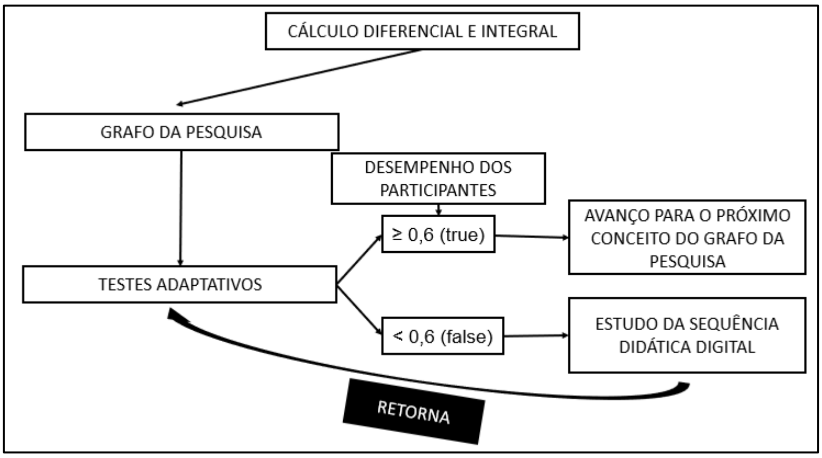
METHODOLOGICAL COURSE

The focus of the research was qualitative, with the adoption of observations, interviews, questionnaires and analysis of the mistakes and successes of the students throughout the adaptive tests. According to Bicudo (2012, p. 17) the qualitative method is “a way of proceeding that allows highlighting the subject of the process, not seen in isolation, but socially and culturally contextualized”. This research was approved by the Human Research Ethics Committee, CAAE: 17226119.6.0000.5349.

A CID was implemented (developed, applied and evaluated) through DDS in the SIENA system, with 7 students who had already attended the Differential to Integral Calculus I course and who had difficulties in understanding the concepts, showing interest in revisiting such theme. It is important to point out that the research was developed during the COVID-19 Pandemic, so the analysis of only 7 students is presented. This investigation is a continuation of the research by Silva (2019), where the graph (implemented in SIENA) and the question banks of the Adaptive Tests with the Derivatives theme were developed. In continuation of this research, DDS were developed, organized according to Filatro's CID (2008), for each concept of the Derivatives thematic graph.

For the development of the experiment, the students received a login and password to access SIENA and carry out the Adaptive Tests of each concept of the graph, and if the performance was less than 0.6 in the [0, 1] interval, the study of the DDS and, after the study, the students should redo the Adaptive Tests until they reach a performance greater than or equal to 0.6. In Figure 11, it is possible to observe the route planned for students in the SIENA System.

Figure 11 - Scheme of research activities



Source: Santos (2021, p. 123).

Data collection was carried out using: Questionnaire to outline the profile of the class/students (age, course, semester, disciplines); analysis of the Siena System database, with the results of the Adaptive Tests before and after the study of the developed sequences; instrument for collecting feedback from students participating in the experiment on the DDS.

ANALYSIS AND RESULTS

Of the seven research participants, students of the Exact Sciences Area, six participants were aged between 20 and 25 years old and 1 between 25 and 30 years old. Five lived in the metropolitan region of Porto Alegre and two in the interior of the state of Rio Grande do Sul, in the cities of Minas do Leão and São Vendelino. As for the workload, 2 people answered that they worked an average of 8 hours a day (1 chemical technician and another administrative manager), 2 people worked an average of 6 hours a day (1 elementary school teacher and 1 kindergarten teacher) and 3 people reported that they were students (3 Mathematics-Bachelor students). Of the investigated participants, 3 had completed higher education (1 is a graduate of the Chemical Engineering course, 1 of Mechanical Engineering and 1 Mathematics-Licenciatura), and one had the Civil Engineering course and was studying Mathematics-Licenciatura, and the others 4 participants were all studying Mathematics-Licenciatura.

Regarding the level of difficulty in Basic Education concepts and procedures (Elementary and High School), on a scale of 1 for little difficulty and 5 for great difficulty, the answers were: 4 answers (1) little difficulty; 2 answers (2) reasonable difficulty; 1 answer (3) medium difficulty.

Regarding the level of difficulty of the concepts of Differential and Integral Calculus, the answers were: 1 answer (1) little difficulty; 2 answers (3) medium difficulty; 2 answers (4) high level of difficulty; 2 Answers (5) Very difficult.

When asked about failures in graduation, 3 participants answered that they had not failed any discipline, 2 participants reported that they had failed one discipline, 1 participant reported that he had failed 2 disciplines, and 1 participant reported that he had failed one 4 disciplines.

Data analysis regarding the performance of each student, in each concept of the graph, is available in Table 1, which contains the grades achieved on the individualized map, available in the SIENA system database, as well as how many times each student performed the test.

Table 1 - Student performance separated by concept

Student	**Basic Mathematics - Arithmetic**	**Basic Mathematics - Algebra**	**Mathematics - Functions**	**Direct Derivatives, Product and Quotient Rule**	**Derivatives - Chain Rule**	**Applications of Derivatives with Problem Solving.**
1	0.998	0.997	0.994	0.993	0.997 0.130	0.991
2	0.983	0.997	0.745	0.993 0.001	0.978 0.002 0.004 0.005	0.996
3	0.958	0.992	0.994	0.935 0.028	0.995 0.066 0.002	0.999

					0.000	
4	0.997	0.999	0.997	0.903	0.995	0.995
				0.008	0.172	0.046
					0.059	
5	0.994	0.999	0.991	0.992	0.978	0.933
				0.597	0.000	0.000
						0.012
6	0.999	0.993	0.997	0.993	0.990	0.996
7	0.991	0.613	0.953	0.945	0.957	0.964
			0.405	0.270		

Source: Research Data (2021).

It is observed that only one student presented medium difficulty in one of the Basic Mathematics concepts. This student obtained a score of 0.405 the first time he took the Basic Mathematics - Functions concept test, after failing the test, the SIENA system provides the DDS on the concept of Functions and, after studying the repetition of the test, the student was approved with a grade 0.953. The student had difficulties performing interpretations of the functions, such as finding the zero(s) of the function, the vertex of the quadratic function, as can be seen in Figure 12.

Figure 12 - Example of a wrong question involving a 2nd degree function

O saldo de uma conta bancária é dado por $S = t^2 - 11t + 24$, onde S é o saldo em reais e t é o tempo em dias. Qual é o valor do saldo mínimo? E em que dias ocorrerá?

a) Saldo mínimo: R$ 6,25 durante o 6º dia.
b) Saldo mínimo: R$ 6,25 durante o 4º dia. **(resposta do aluno)**
c) Saldo mínimo: R$ 6,25 durante o 5º dia. **(resposta certa)**
d) Saldo mínimo: R$ 5,50 durante o 6º dia
e) Saldo mínimo: R$ 5,50 durante o 5º dia

Source: Research Data (2021).

In this question, the student got the minimum balance value right using the value of $y_v = \frac{-\Delta}{4a}$, but when finding the value of $x_v = \frac{-b}{2a}$, the student calculated the wrong value and marked that the balance would occur on the 4th day when the correct answer would be during the 5th day.

In relation to the three concepts related to Derivatives, it can be observed that the students failed and that they studied the DDS. In the concept of Direct Derivatives, Product Rule and Quotient, five students failed the first test, after carrying out the DDS study they took the test again and all were approved. In the Derivatives – Chain Rule concept, five students again had difficulties and failed when taking the test and, of these students, three had to take the test more than once.

In the concept of Direct Derivatives Rule of the Product and Quotient it was possible to observe difficulties mainly in the questions that involved the product or the quotient of functions. In Figure 13, it is possible to verify that the student had difficulties in deriving the functions $f(x)$ and $g(x)$ e, and, later, writing the derivative of the quotient.

Figure 13 - Example of error in the Quotient Derivation

Marque a resposta certa para a derivada de $$y = \frac{\sqrt{x^5}}{4x+7}$$
a) $\frac{dy}{dx} = \frac{3\sqrt{x^5}+\frac{35}{2}\sqrt{x^3}}{16x^2+56x+49}$
b) $\frac{dy}{dx} = \frac{6\sqrt{x^3}+\frac{35}{2}\sqrt{x}}{16x^2+49}$
c) $\frac{dy}{dx} = \frac{6\sqrt{x^5}+\frac{35}{2}\sqrt{x^3}}{16x^2+49}$
d) $\frac{dy}{dx} = \frac{6x^3-4\sqrt{x^3}}{16x^2+56x+49}$ **(resposta do aluno)**
e) $\frac{dy}{dx} = \frac{6\sqrt{x^3}+\frac{35}{2}\sqrt{x^3}}{16x^2+56x+49}$ **(alternativa correta)**

Source: Research Data (2021).

In the concept Derivatives - Rule of the chain it was possible to observe that the students had difficulty in understanding the composite function so that they could derive the functions, using $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, in addition to the difficulties in applying the Derivation techniques, as can be seen in Figure 14, where the student did not derive the exponent of the function.

Figure 14 - Example of error in Derivatives of Composite functions

Qual é a derivada da função: $$y = e^{\frac{1}{x}} + \frac{1}{e^x}$$
a) $y' = \left(e^{\frac{1}{x}}\right) - e^{-x}$ **(resposta do aluno)**
b) $y' = \left(-\frac{e^{\frac{1}{x}}}{x^2}\right) - e^{-x}$ **(alternativa correta)**
c) $y' = \left(-\frac{e^{\frac{1}{x}}}{x^2}\right) + e^{-x}$
d) $y' = \left(-\frac{e^{\frac{1}{x^2}}}{x^2}\right) - e^{-x}$
e) $y' = \left(e^{-\frac{1}{x^2}}\right) - e^{-x}$

Source: Research Data (2021).

Regarding the concept of Applications of Derivatives with Resolution of Problem Situations, it can be seen that two students failed when taking the test, and one was approved after studying the sequence and the other repeated the test more than once. In Figure 15, it is possible to observe that the student did not derive the function correctly to then find the marginal cost when $x = 50$.

Figure 15 - Example of Error in Derivative Problem Solving

Suponhamos que $C(x)$ seja o custo total de fabricação de x pares de calçados da marca WW dado pela equação $C(x) = 110 + 4x + 0{,}02x^2$. Determinar o custo marginal quando $x = 50$.
a) 5
b) 360
c) 214 **(Resposta do aluno)**
d) 2
e) 6 **(Resposta certa)**

Source: Research Data (2021).

After carrying out the tests, the students had access to the materials developed in the DDS and sent feedback with an evaluation of the DDS.

Participant 4's feedback: “The sites are organized in a simple and clear way and helped me a lot with questions regarding Derivatives. I always had doubts about these contents and from the explanatory videos of the study material I was able to understand some concepts that were important for me to get approval. I saw potential in the study material that could be used in calculus classes or workshops, and despite not having used it, I liked the material developed for Basic Education, as it could already be worked with students throughout Elementary School and High school. In general, I consider the material well prepared and developed not only for the tests that were carried out, but to actually be used in school classes and even in calculus classes.”

Feedback from student 5: “The materials are organized in a very explicit way and without much pollution of information around. I liked the way in which the contents are addressed, because in addition to the website being intuitive and directing the study, you have the option of reading the material with the explanations, listening to the explanation of the material and even listening to another teacher explaining different questions. The videos weren't long and didn't keep “hanging around”, the explanation of the content and the application of the exercises were well directed. Regarding the contents presented, I consider the materials to be easy to learn, focusing on the main points with annotations in some observations.”

FINAL CONSIDERATIONS

The research participants who did not achieve the necessary performance to pass certain concepts, after the DDS study, obtained a satisfactory performance to advance in the tests.

For the development of the didactic material used in the Digital Didactic Sequences, aspects related to the contextualization of mathematical knowledge for an application within the students' area of expertise were considered, as cited by Reis (2001) and Cabral and Baldino (2006). This also corroborates with Cantoral (2013), considering the main configurations highlighted by the authors regarding the difficulties of students in the Differential and Integral Calculus I disciplines. In addition, the recognition of the doubts that are commonly presented by students in the Differential and Integral Calculus I disciplines was also taken into account.

In general, practitioners approved the layout developed to be a standard used. Also, the issue of having an explanatory video of the material available was considered, as it would enhance the learning process of students who still do not have the ability to read mathematical concepts algebraically. There were no reports about the videos available on YouTube, but positive comments were made regarding the video being indicated, as it is understood that if there is difficulty, the student has an additional study tool within the sameenvironment.

DESIGN INSTRUCIONAL NO DESENVOLVIMENTO DE UMA SEQUÊNCIA DIDÁTICA DIGITAL COM A TEMÁTICA DERIVADAS

RESUMO

O presente trabalho é constituído de uma investigação sobre as potencialidades do desenvolvimento de uma Sequência Didática Digital (SDD) com diferentes recursos tecnológicos, e foi aplicado a estudantes da área de Ciências Exatas que cursaram a disciplina de Cálculo Diferencial e Integral I. O objetivo geral da pesquisa foi analisar as contribuições de uma SDD, com a temática Derivadas, visando identificar as dificuldades dos estudantes e ampliar a compreensão dos conceitos e a aplicação dos mesmos em situações problemas para alunos dos cursos da área de Ciências Exatas. A metodologia aplicada é de caráter qualitativo, com o desenvolvimento de um experimento com a participação de 7 estudantes do Ensino Superior. As SDD foram desenvolvidas em sites e disponibilizadas no Sistema Siena. As atividades eram compostas por um material de estudos em PDF, um vídeo explicativo do material de estudos, vídeos complementares disponíveis no YouTube e, em alguns conceitos, objetos educacionais disponíveis no software GeoGebra. As SDD foram baseadas no Design Instrucional Contextualizado. Com base na análise dos dados, a partir do banco de dados do sistema Siena, foi possível identificar o conceito em que os alunos apresentaram maior dificuldade- o de Derivadas – Regra da cadeia. Nos conceitos Matemática Básica – Funções, Derivadas Diretas, Produto e Quociente, Aplicações de Derivadas em Resolução de situações Problemas, diferentes alunos fizeram o estudo das Sequências Didáticas Digitais para a retomada dos conceitos nos quais não atingiram um desempenho satisfatório. Os conceitos de Matemática Básica – Aritmética e Matemática Básica – Álgebra, não foram utilizados durante a pesquisa porque os sete estudantes alcançaram um desempenho esperado no primeiro teste adaptativo realizado. Os participantes da pesquisa consideraram que a interface se mostrou intuitiva, com materiais e recursos considerados pelos participantes como adequados ao estudo.

PALAVRAS-CHAVE: Derivadas. Design Instrucional. Sequência Didática Digital.

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