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Sensitivity Analysis of Hydrocyclone Performance in Mineral Processing Using Monte Carlo Simulation

Análise de Sensibilidade do Desempenho de Hidrociclones em Processamento Mineral Usando Simulação de Monte Carlo

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***Abstract* — The hydrocyclone separator is a processing equipment very used in the mineral industry for size separation of solid particles in ore slurries. It is also an equipment with low concerns about its control for an efficient performance. Traditionally, the monitoring and control of hydrocyclone is done by means of its operational variables as the flowrate Q, the density ρ, and the pressure P of the feed slurry stream of the hydrocyclone. However, the main outcome variable of interest in a hydrocyclone is its "cut-off size", d_{50}, which is the size at which 50% of the solids particles, in average, go to the coarse output (underflow) of the hydrocyclone, and the remaining 50%, in average, go to its fine output (overflow). The vast majority of hydrocyclone applications in mineral processing do not measure d_{50} on-line, because this requires the use of very expensive particle size analyzers. Because of this, hydrocyclones are usually operated in open loop with regards to its cut-off size d_{50}, yet under closed loop control of the operational variables that affect d_{50} as the flowrate Q and the density ρ of the feed slurry stream. Nevertheless, most hydrocyclone applications suffer from bad performance due to the lack of understanding about the quantitative impact of the operational variables on the main outcome variable, the cut-off size d_{50}. In this context, this article intends to quantify the relative impact of the flowrate Q and the density ρ on the cut-off size d_{50} by running Monte Carlo simulation on a mathematical model of the hydrocyclone. The results obtained provide useful knowledge for proper control of hydrocyclone performance.**

***Index Terms* — hydrocyclone, mineral processing, Monte Carlo simulation, size separation.**

***Resumo* — Hidrociclones são equipamentos de processamento largamente usados na indústria mineral para separação por tamanho das partículas sólidas existentes em polpas de minério. Tais equipamentos ainda recebem pouca atenção quanto ao seu controle visando uma operação eficiente. Tradicionalmente, o monitoramento e controle automático de hidrociclones é feito com base em variáveis operacionais, como a vazão Q, a densidade ρ e a pressão P do fluxo de polpa de entrada (alimentação). Entretanto, a principal variável de interesse em um hidrociclone é o seu "tamanho de corte", d_{50}, que é o tamanho em que 50% das partículas sólidas, em média, seguem para a saída grossa (*underflow*) do hidrociclone, enquanto os 50% restantes de partículas, em média, seguem para a saída fina (*overflow*). Na grande maioria das aplicações de hidrociclones em processamento mineral, o tamanho de corte d_{50} não é medido em tempo-real, porque isso demanda o uso de complexos e dispendiosos analisadores de partículas. Como consequência, hidrociclones são geralmente operados em malha aberta, em relação à variável de saída d_{50}, ainda que existam controles em malha fechada para as demais variáveis operacionais que afetam o d_{50}, como a vazão Q e a densidade ρ da polpa de entrada. Mesmo assim, muitas instalações de hidrociclones apresentam desempenho ruim devido à falta de conhecimento sobre o impacto quantitativo das variáveis operacionais sobre a variável principal de saída, o tamanho de corte d_{50}. Neste contexto, o presente artigo propõe-se a quantificar o impacto relativo da vazão de entrada Q e da densidade de**

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entrada ρ sobre o tamanho de corte d_{50}, usando Simulação de Monte Carlo sobre um modelo matemático de hidrociclone. Os resultados obtidos fornecem informações úteis para aprimorar o controle do desempenho de hidrociclones.

***Palavras-Chave* — hidrociclone, processamento mineral, separação por tamanho, Simulação de Monte Carlo.**

I. INTRODUCTION

MINERAL processing plants make use of several types of size separation (size classification) equipments to segregate ore particles into *coarse* and *fine* partitions, either in the form of bulk solids or slurries [1,2,3]. Size separation is needed due to reasons as: (1) feeding coarse particles back to breakage processes, like crushing and milling; (2) meeting product particle size requirements; or (3) segregating different mineral species present in the ore, to meet chemical content requirements.

In this context, this article presents a performance analysis of hydrocyclone separators – a size separation equipment widely used in mineral processing. The article is organized as follows: Section II introduces two key concepts related to any size separation process: *particle size distributions*, and *partition curves*. Section III presents the hydrocyclone separator, explaining its working principle and operation. A mathematical model of the hydrocycline, the Plitt's model, is also summarized. Section IV proceeds with a sensitivity analysis of hydrocyclone performance by running Monte Carlo simulation on the equipment model. Finally, Section V discuss the results obtained and provide useful insights for proper control of hydrocyclones.

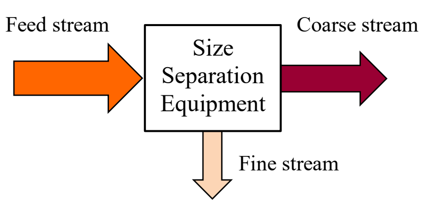

Fig. 1. Diagram of a size separation equipment.

II. QUANTIFYING A SIZE SEPARATION PROCESS

Size separation equipments usually have one input stream (feed) and two output streams (coarse and fine), as shown in Fig. 1. A size separation process is quantified by two key concepts:

- The *particle size distribution* of a *stream*, which corresponds to the cumulative percent amount of solids mass within the particle size range of interest. Fig. 2 shows theoretical examples of particle size distributions, in the size range from 7 to 600 microns, for the feed, coarse, and fine streams of a size separation equipment. Notice that the particle size distribution of the coarse stream has lower amounts of *fine* particles than the feed (and the fine) stream, whereas the fine stream has lower amounts of *coarse* particles than the feed (and the coarse) stream.

- The *partition curve* of a *size separation equipment*, which corresponds to a function that defines how the *particle size distribution* of an output stream results from the *particle size distribution* of the feed stream. The partition curve of an equipment must be referred to a specific output stream, either the coarse or the fine output stream.

Fig. 3 shows three theoretical examples of partition curves. The black curve is an "ideal" partition curve, for which there is a "perfect" separation at a hard *cut-off size* d_H. In Fig. 3, d_H = 90 µm. All solid particles with sizes lower or equal than d_H (fine particles) will go to the fine output stream of the separation equipment, whereas all particles with sizes greater than d_H (coarse particles) will go to the coarse output stream. In contrast, the blue curve is a theoretical curve for a *real* separation equipment, with regards to its coarse stream. The size separation performed by a real equipment is not perfect: some amount of *fine* particles will unduly go to the *coarse* stream (indicated by the bottom shadowed area); and some amount of *coarse* particles will unduly go to the *fine* stream, so that the probability that coarse particles will go to the *coarse* stream increases gradually with the particle size (indicated by the upper shadowed area). Because of this, the partition curve of a real equipment typically has a "slope" or "sharpness", and

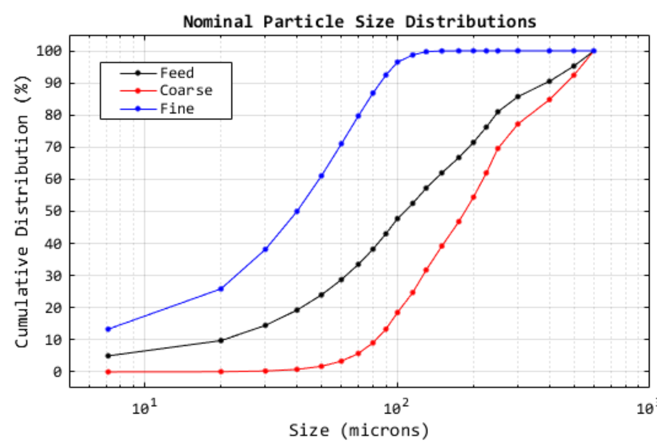

Fig. 2. Particle size distributions in a size separation equipment.

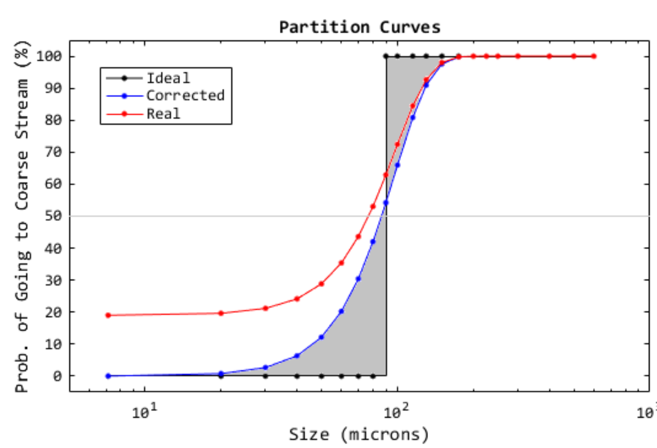

Fig. 3. Examples of theoretical partition curves.

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its cut-off size is defined as the size at which 50% of the solids mass from the feed stream goes to the coarse stream (since the partition curve is referred to this output stream), and it is termed as d_{50}. The closer to vertical is the slope, the higher is the separation efficiency. Additionally, in a real equipment, a fixed amount of *fine* particles is unduly "dragged" to the *coarse* stream, and this is termed as "short-circuit" or "by-pass". In the real partition curve in Fig. 3, the short-circuit is 19.0%. The blue curve in Fig. 3 is termed as the "corrected" partition curve, obtained by disregarding the by-pass effect. Both the "slope" and the "by-pass" represent imperfections in the separation process of a real equipment. The 50% cut-off size in the corrected partition curve is denoted as d_{50c} and is always higher than d_{50} in the corresponding real partition curve. Since the difference between the real and the corrected partition curves is due to the "by-pass", the performance of a size separation equipment can be fully specified by either its real or corrected partition curve (d_{50} or d_{50c}) and its "by-pass".

Notice that the particle size distribution and the partition curve are distinct concepts, although they share the same quantitative representation, as shown in Fig. 2 and Fig. 3. The particle size distribution is the cumulative distribution of solid particles in a *stream*, whereas the partition curve represents a "probabilistic" aspect of the size separation process performed by an *equipment*. The particle size distributions of the coarse and the fine streams will depend not only on the equipment performance (its partition curve), but also on the particle size distribution of the feed stream. However, no matter is the shape of a partition curve, it will always have a 50% cut-off size, d_{50}.

III. The Hydrocyclone Separator

A particular type of size separation equipment for processing of ore slurries is the hydrocyclone separator, or simply hydrocyclone [1,2,3,4], shown in Fig. 4. It has a cylindrical body which becomes conical downwards, an input nozzle (feed inlet), a bottom output nozzle called *apex* or *spigot*, whose stream is termed as *underflow*, and an upper output nozzle whose stream is termed as *overflow*. The hydrocyclone has no moving parts, although there are pneumatic-actuated apex nozzles with variable aperture.

The pressurized stream entering the feed inlet is provided by a slurry pump. The input nozzle is tangential to the hydrocyclone body, creating two rotational flows (vortexes) within the hydrocyclone: an outer vortex that goes to the apex and carries most of the coarse particles downwards to the underflow, and an inner vortex that carries most of the fine particles upwards to the overflow. The size separation between coarse and fine particles is due exclusively to differences in the centrifugal forces acting on the solids particles according to their sizes and densities. The hydrocyclone does not need any kind of energy to perform the separation process, so that it is a *passive* separation equipment.

Hydrocyclones are widely used in mineral processing due to their favorable capacity-to-size ratio and reasonable low maintenance [3]. As a size separation equipment, the performance of a hydrocyclone can be defined by its partition curve. The main goal of a hydrocyclone is to produce an underflow with most of the coarse particles and considerably less fine particles than the feed stream, according to its designed cut-off size d_{50}. The cut-off size is an indicator of the separation performance of a hydrocyclone, and is the ultimate variable to be controlled in a hydrocycloning process. Therefore, a quantitative understanding on how d_{50} changes in response to variations in the operational conditions of the hydrocyclone can be obtained if a mathematical model of the equipment is available. Before discussing a hydrocyclone model, one needs to understand the parameters and variables involved in a hydrocycloning process.

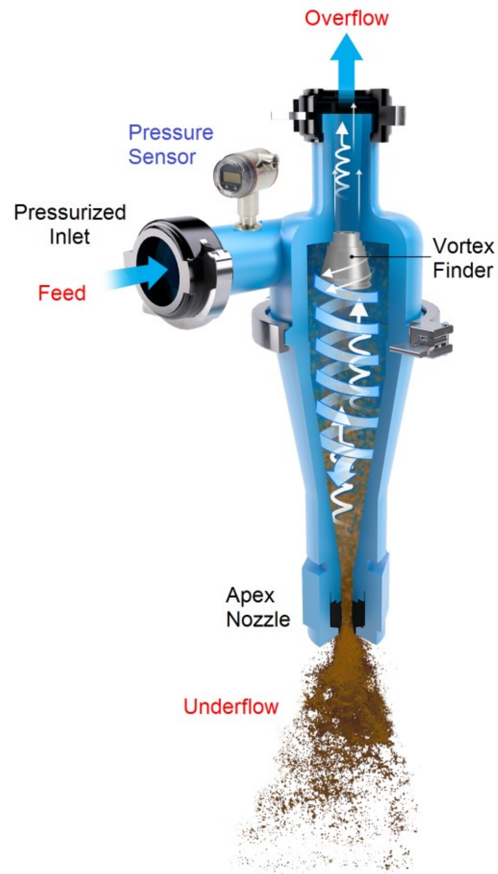

Fig. 4. Illustration of a hydrocyclone separator.

Fig. 5 shows a typical layout of a hydrocycloning process for mineral processing applications. A slurry tank receives an input stream of raw concentrated slurry from an upstream processing unit, and a dilution water stream so that the density (or solids concentration) of the pumped output stream from the tank can be lowered to a desired value. The diluted slurry pumped from the tank feeds a *hydrocyclone cluster* (or *hydrocyclone battery*), shown in Fig. 6, which is simply a set of hydrocyclones arranged in a common mechanical unit. The number of hydrocyclones in a cluster is defined during its design, as the ratio between the maximum pumped slurry flowrate Q_{Pmax} and the maximum feed flowrate for a single hydrocyclone Q_{Fmax}. As an example, a cluster of hydrocyclones with Q_{Fmax} = 90 m^3/h designed to operate with a maximum pumped flowrate Q_{Pmax} =

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650 m^3/h requires Q_{Pmax}/Q_{Fmax} = 650/90 ≈ 7.22 ≈ 8 hydrocyclones. The number is rounded up, otherwise Q_{Pmax} is not satisfied.

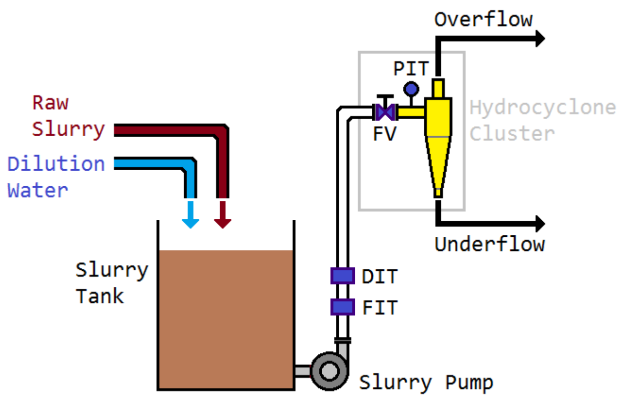

Fig. 5. Typical layout of a hydrocycloning process.

Fig. 6. Example of a hydrocyclone cluster for mineral processing. (Source: WEIR Minerals, Inc)

The basic instrumentation necessary for monitoring and control of hydrocyclones comprises: a volumetric flow instrument (FIT) and a density instrument (DIT) in the feed pipeline; a pressure instrument (PIT) close to the hydrocyclone inlet; and an on-off valve at the feed inlet. The most commonly used technologies for those instruments are: magnetic flowmeters [11], radioactive density meters [12,13], flanged seal pressure meters [14], and knife-gate valves [15]. Such instrumentation allows the measurement of the feed volumetric flowrate Q, the feed density ρ, and the input pressure P of the hydrocyclone. The inlet valve is used only to release or block the hydrocyclone for operation. In a hydrocyclone cluster, it is not necessary to install one pressure meter at each hydrocyclone inlet. A single pressure meter can be installed at the feed chamber of the cluster, where the pressure is virtually equal to the inlet pressure of all hydrocyclones in the cluster.

A hydrocycloning process must ensure that the cut-off size d_{50c} (or d_{50}, when considering the real partition curve) meets a given requirement, for which the hydrocyclones were designed. It is necessary to keep d_{50c} stabilized from closed loop control of the variables Q, ρ, and P. The density ρ is controlled by the water addition to the slurry tank. The input flowrate Q is controlled by manipulating the rotating speed of the slurry pump. The pressure P is mainly a consequence of the input flowrate, and should be manipulated by opening or closing the on-off valves of the hydrocyclones in the cluster.

How much does the cut-off size d_{50c} changes in response to variations in the operational variables Q and ρ? In other words, what is the *sensitivity* of d_{50c} regarding to those variables? Understanding this sensitivity is fundamental for proper design of control systems for hydrocyclones. The lack of knowledge about the sensitivity of d_{50c} regarding to Q and ρ leads to poor design and tuning of control loops to regulate those variables for open loop control of d_{50c}. The goal of this article is to quantify the sensitivity of d_{50c} regarding to those operational variables by using a mathematical model of the hydrocyclone, providing conclusions to improve the closed loop control of the operational variables that define the open loop control of d_{50c}.

The cut-off size of a hydrocyclone depends on two class of parameters [2], summarized in Table I:

1) *Constructive or Design Parameters*
   Parameters related to physical aspects of the hydrocyclone, mainly its geometry. This include its inlet diameter, body diameter, apex diameter, overflow outlet diameter, and free vortex height. Those are *fixed parameters*, however, one exception is the pneumatic apex with variable diameter.

2) *Operational parameters*
   Parameters related to process variables that set the operational condition of the hydrocyclone. Usually, those are *varying parameters* related to properties of the slurry stream processed by the hydrocyclone, such as: particle size distribution and density (or solids concentration) of the feed slurry, feed flowrate and pressure, and solids density.

TABLE I

CONSTRUCTIVE AND OPERATIONAL PARAMETERS OF A HYDROCYCLONE

Parameter	Notation
Constructive parameters	
Cylindrical body diameter	D_C
Inlet diameter	D_F
Overflow diameter	D_O
Underflow diameter	D_U
Free vortex height	H
Operational parameters	
Liquid density	ρ_L
Solids density	ρ_S
Feed slurry density	ρ_F
Feed solids concentration by volume	Cv_F
Feed solids concentration by mass	Cm_F
Feed flowrate	Q_F
Feed pressure	P
By-pass (short circuit) fraction	α
Outcome variable	
Corrected cut-off size	d_{50c}
Cut-off size	d_{50}

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In past decades, some empirical models of the hydrocyclone were developed using experimental data and theoretical knowledge on fluid mechanics. The high complexity involving modeling the rotational flows in a hydrocyclone and the interactions between the solids particles and the liquid, impose limitations for the development of an exact mathematical model of the hydrocyclone. Among the currently available models of the hydrocyclone, the models due to Lynch & Rao [7], Plitt [8], and Mullar & Jull [9] have been the most used in the design and analysis of hydrocyclones.

In this work, the Plitt's model of a hydrocyclone was used for the sensitivity analysis of the cut-off size d_{50c} with regards to the operational parameters of a hydrocyclone with fixed constructive parameters. Table II summarizes the constructive and operational parameters of a real hydrocyclone [16] considered in this work.

In the 1970's, L. R. Plitt developed a mathematical model of the hydrocyclone [8] using a set of experimental data from 300 experiments with hydrocyclones, some of which were performed by the author, and some were performed by other researchers and previously published in the literature. The Plitt's model assumes that the *corrected partition curve* of a hydrocyclone has the form of a Rosin-Rammler equation [5,6]:

$$Y_c(d) = 1 - e^{-0.6931\left(\frac{d}{d_{50c}}\right)^m} \quad (1)$$

where d (μm) is the size fraction of interest, d_{50c} (μm) is the corrected cut-off size, and m is the "slope" or "sharpness" of the partition curve, given by:

$$m = 1.93 e^{-1.58 R_v} \left(\frac{D_c^2 H}{Q} \right)^{0.15} \quad (2)$$

The corrected cut-off size d_{50c} (μm) is defined as:

$$d_{50c} = 52.5 \frac{D_C^{0.46} D_I^{0.6} D_O^{1.21} e^{0.063 Cv_S}}{D_U^{0.71} H^{0.38} Q^{0.45} (\rho_S - \rho_L)^{0.5}} \quad (3)$$

The pressure drop P (KPa) and the feed flowrate Q (l/min) are defined as:

$$P = 1.88 \frac{e^{0.0055 Cv_S}}{D_C^{0.37} D_I^{0.94} H^{0.28} (D_U^2 + D_O^2)^{0.87}} Q^{1.78} \quad (4)$$

$$Q = 0.3251 \frac{D_C^{0.208} D_I^{0.528} H^{0.157} (D_U^2 + D_O^2)^{0.489}}{e^{0.0031 Cv_S}} P^{0.562} \quad (5)$$

As an example, the corrected partition curve (blue curve) shown early in Fig. 3 was obtained from equation (1), for specific values of constructive and operational parameters.

According to equation (3), the operational parameters that affect directly the corrected cut-off size d_{50c} (and also the real cut-off size d_{50}) are the feed flowrate Q, the feed solids concentration by volume Cv_F, and the solids specific gravity ρ_S. Assuming a hydrocyclone with fixed constructive parameters, its cut-off size d_{50c} will be theoretically fixed, and the performance of its separation process will be due to the operational variables Q, Cv_S, and ρ_S. Hence, for practical applications, it is important to understand the sensitivity of the cut-off size d_{50c} regarding to Q, Cv_S, and ρ_S. This is discussed in the next section.

TABLE II
PARAMETERS OF A REAL HYDROCYCLONE USED FOR SENSITIVITY ANALYSIS

Parameter	Value
Plitt's model hydrocyclone parameters	
Cylindrical Body diameter	D_C = 26" = 66.40 cm
Inlet diameter	D_F = 10" = 25.40 cm
Overflow diameter	D_O = 12" = 30.48 cm
Underflow diameter	D_U = 6" = 15.24 cm
Free vortex height	H = 273.50 cm
Fluid (bauxite ore slurry)	
Liquid density (water)	ρ_L = 1.0 g/cm^3
Solids density (**bauxite ore**)	ρ_S = 2.7 g/cm^3
Feed slurry %Solids by volume	Cv_F = 14.5 %
Feed slurry %Solids by mass	Cm_F = 30.0 %
Feed slurry density	ρ_F = 1.23 g/cm^3
Nominal operating condition	
Feed flowrate	Q_F = 545.0 m^3/h
Feed %Solids by mass	Cm_F = 30.0 %
Pressure	P = 75.8 KPa
Variability	
Feed flowrate	ΔQ_F = ±5 % to ±35 %
Feed slurry %Solids by volume	ΔCv_F = ±5 % to ±35 %
Solids density	$\Delta \rho_S$ = ±5 % to ±35 %

IV. SENSITIVITY ANALYSIS

In a process control context, understanding how much the cut-off size d_{50c} of a hydrocyclone is affected by variations in its operational parameters Q, Cv_S, and ρ_S, around a nominal operating point $\{Q^0, Cv_S^0, \rho_S^0\}$, allows us to understand *what should be the permissible amount of variability in the operational variables that do not lead to a bad performance*.

A sensitivity analysis can be done in either an analytical or a numerical way. An analytical evaluation requires computing the derivatives of d_{50c} in equation (3) with regards to the operational variables of interest. On the other hand, a numerical analysis, using Monte Carlo simulation, simply uses equation (3) to compute the statistical distribution of d_{50c} given simulated distributions of the operational variables of interest.

In this work, the sensitivity analysis of d_{50c} was performed using Monte Carlo simulation, using the Plitt's model of the hydrocyclone, but using the parameters of a real equipment [16]. The analysis was done with the following steps:

1) Choose one or more of the variables Q, Cv_S, and ρ_S to be varied.
2) Generate a set of N=10,000 random values for the variable(s) under a normal distribution with mean and variability according to Table II.

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3) Apply the mean (nominal) values to the hydrocyclone model to compute the nominal cut-off size d_{50c}.
4) Apply the random values to the hydrocyclone model to compute the resulting values for the cut-off size d_{50c}.
5) Quantify the sensitivity of d_{50c} regarding the variables using the Pearson's Coefficient of Variability.
6) Identify the implications of the sensitivity of d_{50c} in a process control context.

The results of the sensitivity analysis are described in the following.

A. Sensitivity to the feed flowrate Q_F

The first sensitivity analysis took into account the feed flowrate Q_F. A set of 10,000 random values for Q_F were generated under a normal distribution with average (nominal) value of 545.0 m^3/h and a variability of ±20%, according to Table II. The resulting partition curves are shown in blue lines in Fig. 7 (the lines are superimposed) along with the partition curve for the nominal condition (black line). The corresponding variation in the cut-off size d_{50c} is shown in the histogram in Fig. 8. It was found that the variation of ±20% in Q_F leads to a variation of around ±3.93% in d_{50c}. This suggests that the feed flowrate Q_F has a little impact on d_{50c}. However, Q_F should be controlled by a well designed and tuned control loop because its impact on d_{50c} is not negligible.

B. Sensitivity to the feed solids concentration Cv_F

The second sensitivity analysis was regarding the feed solids concentration by volume Cv_F, according to the nominal condition and the variability shown in Table II. The resulting partition curves are shown in Fig. 9. The corresponding variation in the cut-off size d_{50c} is shown in the histogram in Fig. 10. It was found that the variation of ±20% in the feed solids concentration Cv_F leads to a variation of around ±7.88% in d_{50c}. This indicates that Cv_F has a greater impact on d_{50c} compared to Q_F. In a process control context, this means that the density of the feed stream to the hydrocyclone must be very well controlled around the nominal value, to prevent excessive variations that will have a major impact on the cut-off size d_{50c} of the hydrocyclone.

TABLE III
SENSITIVITY OF THE CUT-OFF SIZE d_{50c} TO VARIATIONS IN THE OPERATIONAL PARAMETERS

Variation in operational variables Q_F, Cv_F, or ρ_S	Variation in d_{50c} from the variation in Q_F	Variation in d_{50c} from the variation in Cv_F	Variation in d_{50c} from the variation in ρ_S
0 %	0 %	0 %	0 %
5 %	0.97 %	1.96 %	1.71 %
10 %	1.94 %	3.93 %	3.45 %

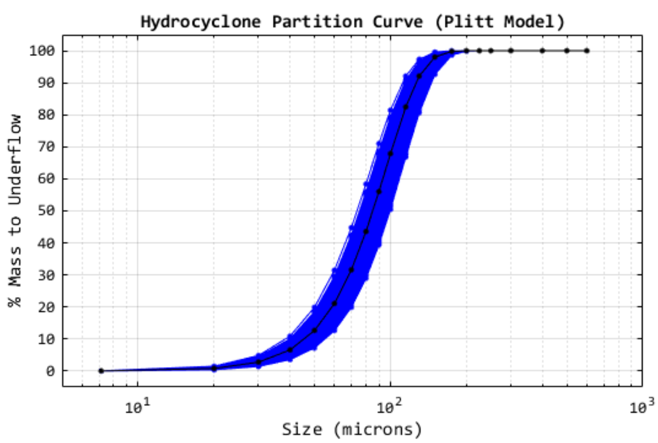

Fig. 7. Partition curves for 20% variation in Q_F.

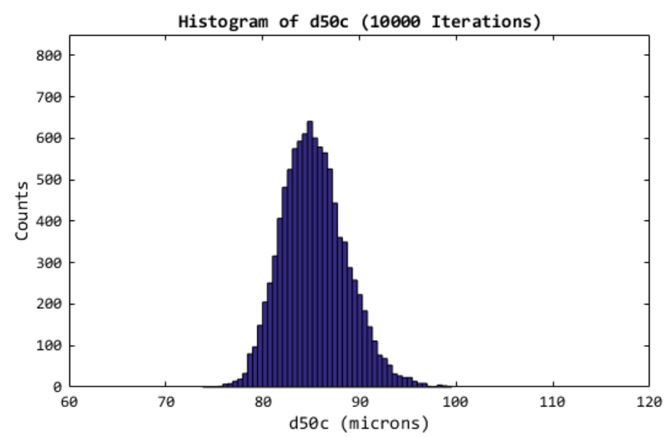

Fig. 8. Histogram of d_{50c} for 20% variation in Q_F

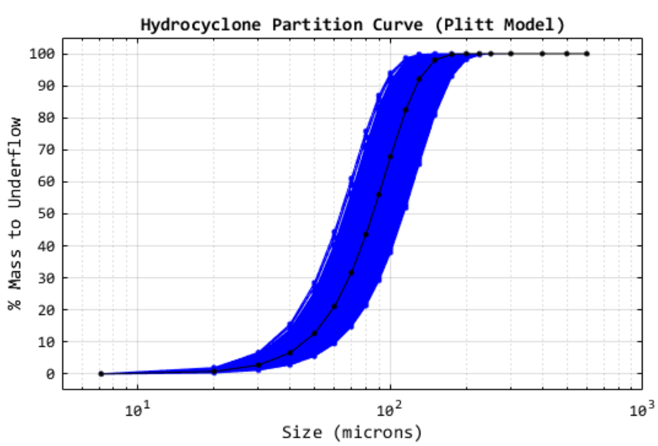

Fig. 9. Partition curves for 20% variation in Cv_F.

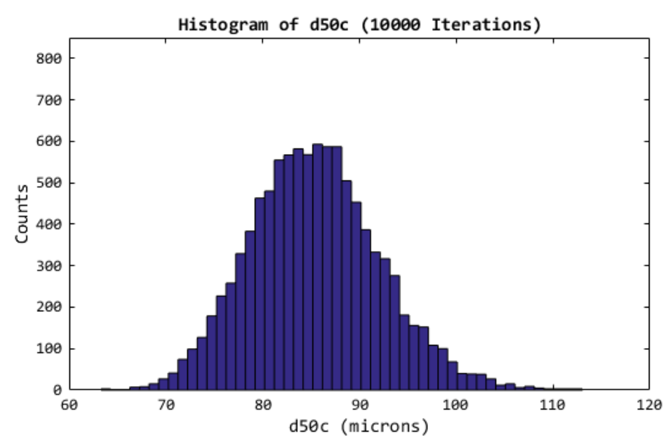

Fig. 10. Histogram of d_{50c} for 20% variation in Cv_F.

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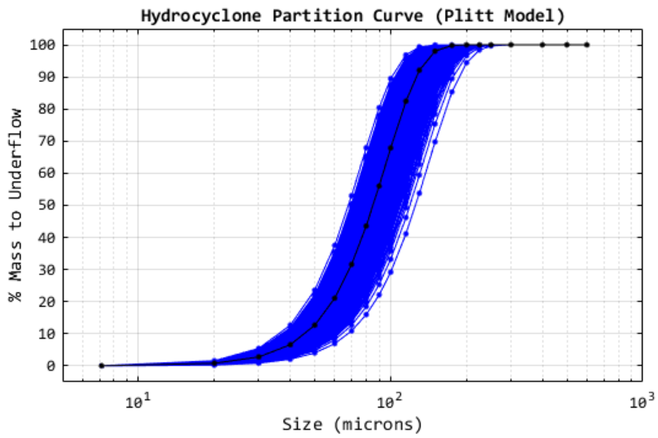

Fig. 11. Partition curves for 20% variation in ρ_S.

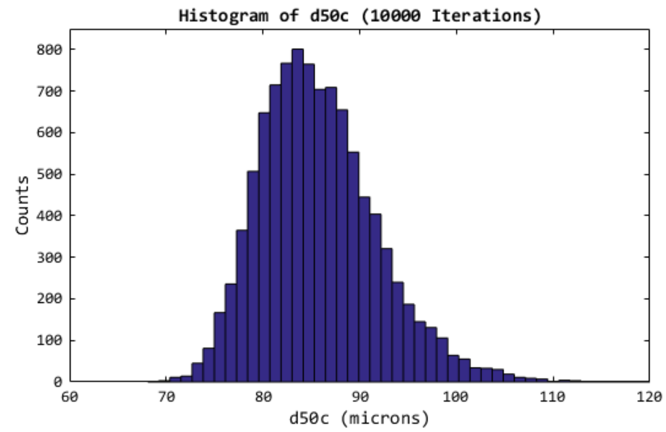

Fig. 12. Histogram of d_{50c} for 20% variation in ρ_S.

C. Sensitivity to the solids density ρ_S

The solids density ρ_S was considered in the third sensitivity analysis. The resulting partition curves and histogram are shown in Fig. 11 and Fig. 12. The variation of ±20% in the solids density ρ_S leads to a variation of ±7.12% in d_{50c}. This indicates that ρ_S has a high impact on d_{50c}. In a process control context, this means that ρ_S should be well controlled. However, the solids density of an ore is due to its geological characteristics that cannot be controlled by automation systems. Therefore, the variations in ρ_S should be regarded as disturbances in the process.

Table III shows the variations in d_{50c} corresponding to specific variations from ±5% to ±35% in the operational parameters Q_F, Cv_F, and ρ_S. For better understanding of the sensitivity of d_{50c} regarding each operational parameter, those values are show in Fig. 13.

V. Discussion of the Results

The sensitivity results shown in Fig. 13 clearly indicate the operational parameters of a hydrocyclone which have major impact on its cut-off size d_{50c}. The feed solids concentration Cv_F and the solids density ρ_S are such highly impacting parameters. In a real hydrocycloning process, the observed variabilities in the operational parameters can be used in a sensitivity analysis to compute the resulting variability of d_{50c}. This variability can be compared to the desired performance of the hydrocyclone, in order to identify which operational parameters may need better control.

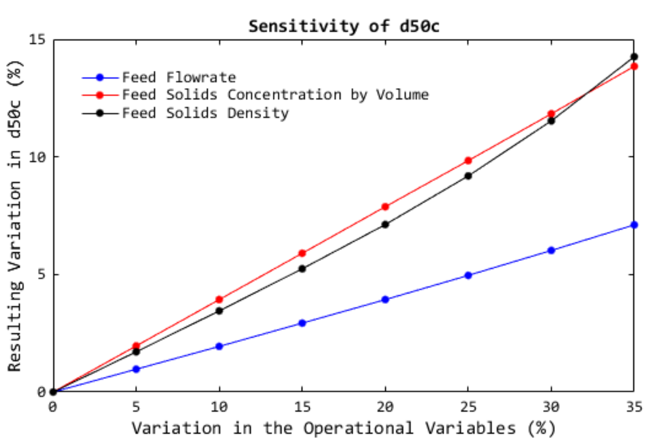

Fig. 13. Sensitivity of the cut-off size d_{50c} to variations in the operational parameters of the hydrocyclone.

Suppose that the hydrocycloning process from Table II requires that d_{50c} has less than ±6% variation, measured by the Pearson's Variability Coefficient. Then, from Table III, one should prevent, for example:

- A variation of ±20% in Cv_F, which leads to a variation of ±7.88% in d_{50c}.
- A variation of ±20% in ρ_S, which leads to a variation of ±7.12% in d_{50c}.
- A variation of ±35% in Q_F, which leads to a variation of ±7.11% in d_{50c}.

Notice that the results from Table III and Fig.13 were obtained from variation in one operational parameter. In practice, those parameters vary independently, and the effects of their variations will be added, causing an even greater impact on the cut-off size d_{50c} of the hydrocyclone. This is another reason to invest in well designed and tuned control loops for the operational parameters.

The solids density ρ_S cannot be controlled, since it is an intrinsic property of the solids, and should be considered as an unmeasured disturbance in the process.

VI. Conclusion

The results obtained in this work clearly indicate the important to assess the sensitivity of the cut-off size d_{50c} of a hydrocyclone regarding to its operational parameters. In spite this work considered only the feed flowrate, feed solids concentration, and solids density, other operational parameters can be taken into account in the sensitivity analysis of d_{50c}, according to the hydrocyclone model being used. As a suggestion for further works, the sensitivity analysis could be extended to other hydrocyclone models.

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