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Mathematical modeling as a context for the professional development of prospective mathematics teachers

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ABSTRACT

This article aims to identify elements associated with professional development in the context of a mathematical modeling subject in prospective teachers' education process. Thus, the data that constitute our analysis material were produced in the context of this subject, which aimed to discuss theoretical concepts that characterize mathematical modeling, debate about modeling, and allow prospective teachers to develop mathematical modeling activities. In analyzing the data, we consider the productions of three prospective teachers, on which we make our inferences about mathematical modeling as a promoter of professional development. As a result, the prospective teachers recognize that learning about modeling enables them to create reflective and participatory learning environments. They also highlight some characteristics, such as being an eternal student and having flexibility and mastery of mathematical content, which are related to professional development. Furthermore, they seem to understand that teacher education is a continuous process that demands reflection and adaptation and that specific and pedagogical knowledge must be integrated for it to continue being completed.

KEYWORDS: Professional Development; Mathematics Education; Initial Teacher Education; Mathematical Modeling

Modelagem matemática como contexto para o desenvolvimento profissional de futuros professores de matemática

RESUMO

O presente artigo tem como objetivo identificar elementos associados ao desenvolvimento profissional no contexto de uma disciplina de modelagem matemática na formação de futuros professores. Os dados que configuram nosso material de análise foram produzidos no contexto dessa disciplina, que tinha como proposta discutir conceitos teóricos que caracterizam a modelagem matemática, debater sobre o fazer modelagem e proporcionar que os futuros professores desenvolvessem atividades de modelagem matemática. Na análise dos dados consideramos as produções de três futuros professores nas quais investigamos a modelagem matemática como promotora de desenvolvimento profissional. Como resultados, os futuros professores reconhecem que aprender sobre modelagem os capacita a criar ambientes reflexivos e participativos de aprendizagem, além de destacarem características como ser um eterno estudante, ter flexibilidade, ter domínio dos conteúdos matemáticos. Além disso, parecem compreender que a formação docente é um processo contínuo que demanda reflexão e adaptação e que para que ela siga se completando há necessidade de integração de conhecimentos específicos e pedagógicos.
PALAVRAS-CHAVE: Desenvolvimento Profissional; Educação Matemática; Formação Inicial do Professor; Modelagem Matemática.

INTRODUCTION

Since the 1980s, several authors have made efforts to discuss mathematical modeling (MM) in mathematics education. Among these studies, some investigate MM in association with teacher education (Doerr & Lesh, 2003; Almeida & Dias, 2004; Dias, 2005; Oliveira, 2010; Chaves, 2012; Oliveira, 2016; Kaczmarek & Burak, 2018; Mutti & Klüber, 2021). Our study also follows this direction.

The first investigations focusing on MM and teacher education were carried out in the 2000s (Barbosa, 2001; Almeida & Dias, 2004; Dias, 2005) and some in the 2010s (Klüber, 2012; Pollak & Garfunkel, 2013; Oliveira, 2016), had as one of their interests guiding teachers so that they could use MM in the classroom. During that period, we sought to understand teachers' beliefs about MM and discuss possibilities for its implementation to be present in these teachers' classes. In this research scenario, teachers face difficulties implementing MM in their classes. Therefore, some research focused on investigating them (Bisognin & Bisognin, 2012; Oliveira & Barbosa, 2013; Oliveira, 2010; Ceolim, 2015; Gaston & Lawrence, 2015; Klüber & Tambarussi, 2017).

With the advancement of research that articulates MM and teacher education, other aspects become of interest to researchers, for example, teacher education contexts that go beyond the organization of formative courses or specific formative actions (Rosa, Reis, & Orey, 2012; Wichnoski & Klüber, 2015; Mutti & Klüber, 2018; Oliveira & Klüber, 2018; Martens & Klüber, 2024). Professional development (PD) emerges as a research possibility in this investigative trend. We see PD as a continuous process in which teachers have opportunities for professional emancipation, which involves changes in knowledge, beliefs, skills, and attitudes toward mathematics teaching (Sowder, 2007). Furthermore, for PD, teachers' needs are the outset for formative actions since they are central to the teaching and learning processes (Sowder, 2007).

The scenario for producing data for this investigation was a subject called Mathematical Modeling (MM) from the perspective of mathematics education, with a workload of 60 hours, offered to the fourth-year class of a teaching degree in mathematics at a public university in northern Paraná. To produce the data, we considered the transcripts of the classes, the answers to the questions addressed to the students in some classes, and the *vaivém* (back-and-forth feedback). In the analysis process, we followed Sowder's (2007) guidelines through the objectives for the mathematics teachers' PD to achieve our objective.

Considering our objective, this text is structured on discussions about MM and professional development. Next, we elucidate the context of the research and the methodological aspects, after which we present our analyses and, finally, our reflections on the study developed.

ON MATHEMATICAL MODELING IN TEACHER EDUCATION

The discussions considering interlocutions between MM and teacher education undertaken since the 2000s have focused on initial teacher education (Barbosa, 2004; Braz & Ceolim, 2011; Pollak & Garfunkel, 2013; Malheiros, 2016; Ribeiro & Meneghetti, 2024) and continuing teacher education (Almeida & Dias,

2004; Dias, 2005, Oliveira, 2010; Rosa & Kato, 2014; Anhalt & Cortez, 2015; Tambarussi & Klüber, 2015; Rosa, 2018; Cambi & Caldeira, 2023). In both scenarios, research has pointed to the fact that the implementation of modeling practices is associated with teachers' knowledge of MM and the relationships they establish with it, especially in their formative contexts.

In the context of initial teacher education, Barbosa's research (2001) indicates that teachers recognize the benefits of MM, such as the contribution to understanding mathematical concepts and developing research skills. However, they also point out obstacles, such as educational bureaucracy and students' and parents' reactions. Teachers usually support modeling but feel insecure about implementing it in the classroom. Furthermore, the author recommends that modeling be related to other subjects in initial teacher education, ensuring that prospective teachers complement their education in mathematical and pedagogical aspects.

Barbosa (2004) states what he calls MM cases (Table 1), in which there is variation in MM activities and teachers' and students' actions. The guidelines are that the tasks should vary from the most structured ones to projects in which students must handle the entire process. The characteristics of the activity change students' and teachers' actions; i.e., teachers gradually share with their students the responsibility of conducting MM activities.

Table 1

Tasks in the modeling process

	Case 1	Case 2	Case 3
Problem formulation	Teacher	Teacher	Teacher/Learner
Simplification	Teacher	Teacher/Learner	Teacher/Learner
Data collection	Teacher	Teacher/Learner	Teacher/Learner
Solution	Teacher/Learner	Teacher/Learner	Teacher/Learner

Source: Barbosa (2004).

In this framework, the cases represent guidance for MM practices. They also exemplify the flexibility of modeling in different school contexts, ranging from small research projects, as in case 1, to more extensive projects in cases 2 and 3.

Within continuing education, Dias's (2005) research sought to associate teachers' impressions of mathematics and its teaching with the experiences developed in MM. The result reveals that teachers are excited about MM and its possibilities of approaching real-world topics to teach mathematical content. However, the author underscores that when it comes to using MM in class, teachers feel insecure, arguing that their lack of experience inhibits them. To minimize this possible difficulty, the author contextualizes that MM in teacher education can be seen as an opportunity to "learn" about MM, "learn" through MM, and "teach" using MM.

Pollak and Garfunkel (2013) also emphasize that teachers' active participation in MM activities strengthens their pedagogical skills and builds meaningful professional experiences. They argue that teachers must experience

MM as students to fully understand its general and specific aspects, corroborating the need for continuing education and reflection on educational practices.

In this research scenario —which highlights how important it is that teacher education courses provide contexts that help teachers with MM, enabling them to use it in their teaching practices— other approaches emerge, such as Oliveira's (2010). Within the scope of MM in continuing teacher education, this author investigates the situations in which teachers manifest tensions after using MM in their teaching. In particular, the author discusses and analyzes how they are constituted and how teachers deal with these tensions in the context of MM activities.

Oliveira (2010) identified the tensions that emerged from the choice of the theme, the sequencing of activities, and the pace of their teaching practice. Regarding the theme, the author realized the tension was caused by teachers wanting a topic that did not require content they considered complicated for students. Most of the time, they tried to think of topics close to those they were already used to addressing in their classes. Tensions related to sequencing concerned the sequence of activities the teacher had planned, and, in this sense, it impacted practical issues because, by favoring interactions between students and teachers, MM activities enabled the emergence of issues previously overlooked by the teacher. Tensions related to pacing were associated with the time allocated to each activity since the MM environment can alter the class dynamics and students' involvement in discussing the topic, which, in this sense, took longer than the teacher expected.

These forms of tension somehow manifest teachers' lack of self-confidence in handling MM. They indicate that teachers are cautious in modeling due to their limited knowledge and experience in carrying out that type of activity in class (Barbosa, 2004; Dias, 2005; Oliveira, 2010). This finding points to the need for research that aims to address theory and practice in a more integrated way, in addition to continuing to consider the particularities and individual experiences of teachers.

In an attempt to highlight the trajectory of prospective mathematics teachers who chose to bring MM to basic education classrooms, Malheiros (2016) sought, in the context of initial teacher education, to promote discussions and reflections, which occurred in the elaboration and implementation in schools. The author understands that in the context of MM, experiences provide prospective teachers with an understanding of the possibilities of its use as a pedagogical approach when discussing it, aiming at future teaching practice.

Rosa's (2018) research in the context of continuing education highlights the concept of a mediating teacher and places it as essential for pre-service teachers' PD. It highlights the importance of a formative space that allows teachers to question, analyze, and improve their teaching practices based on previous experiences and teaching theories. It confirms that education processes must include the development of academic skills —related to the specific mathematics content— and methodological skills —which involve the application of differentiated teaching strategies with MM. In the context of research results, Rosa (2018) points out that reflection on teaching practices is essential in the

education process, as is the need to encourage innovation in pedagogical approaches.

In this scenario of teacher reflection, Rosa and Kato (2014) show that due to its characteristics, MM promotes situations that encourage teachers to reflect on their practice. The authors clarify that this occurs because MM requires mastery of mathematical content, creativity, dialogue, and students' active participation, elements that challenge teachers who are used to traditional teaching approaches. The authors also point out that MM encourages teachers to become more reflective professionals, capable of adapting and continually improving their pedagogical practices.

Tambarussi and Klüber (2015) present and reflect on the phenomenon of teacher education in MM with teachers who graduated from the Educational Development Program (EDP) and indicate the many gaps in the education process developed in this program. The debate about these gaps suggests weaknesses in the formative process and in time to reflect on it. Tambarussi and Klüber (2015) say that different reflections are significant so that teacher education in MM can advance and contribute to its permanence in the educational field.

Cambi and Caldeira (2023) investigate conditions that transformed the teacher's role within the MM context. The researchers identified that the teacher's discourse not only created an environment conducive to this change in their role as a teacher but also that modeling was established as a pedagogical alternative for mathematics teaching in this teacher's practice.

In general, these studies show that familiarization with MM during the education process and when taking MM to the classroom contributes to teacher education and can favor changes in their discourses and pedagogical practices. Understanding the sociocultural role of mathematics promoted by MM practices, also in teacher education, cultivates skills in problematizing and investigating real situations. This recognition contributes to MM becoming part of the curriculum and aims to enable students to assume greater autonomy in collecting, analyzing data, and formulating problems.

In this context of understanding that MM encourages in-service and pre-service teachers to reflect on their practices, discussing the knowledge they have of MM, mathematics, and teaching, as well as considering that MM can constitute a scenario for teacher education, we propose to relate MM to the teacher's PD, a theme that we explore below.

ABOUT PROFESSIONAL DEVELOPMENT

From the 1990s onwards, scholars such as Nóvoa (1991) and Schön (1992) discussed teacher education based on the notion of a reflective teacher, in which the education process began to put forward teachers' self-reflections.

New proposals defending the need to overcome a compartmentalized and pragmatic idea of an education process were developed to fill teaching gaps and meet this bias in teacher education. This scenario encourages de debate around PD, supporting reflections on teachers' professional learning.

The respect for teachers' learning processes and the interest in presenting formative methods that differ from traditional perspectives led some authors to present their characterizations of PD (Sowder, 2007; Ponte & Chapman, 2008; Sancar, Atal, & Deryakulu, 2021).

According to Sowder (2007), the needs of (prospective) teachers must be considered as a starting point for formative actions when aiming at their PD. She conceives PD as a continuous process in which teachers have opportunities for professional emancipation, characterized by changes in knowledge, skills, and attitudes toward mathematics teaching. The author also points out that (prospective) teachers' needs should be the starting point for formative actions since proposals for educational changes must be coherent with teachers' realities.

Sowder (2007) presents six objectives to explain what is necessary for teachers to develop professionally, i.e., developing a shared vision for mathematics teaching and learning, developing knowledge of mathematical concepts, developing an understanding of how students think and learn mathematics, developing pedagogical knowledge of the content, developing an understanding of the role of "equity" in school mathematics, and developing a sense of identity as a mathematics teacher.

Developing a shared vision for teaching and learning mathematics relates to the importance of teachers sharing visions for teaching and learning mathematics in collective and social contexts so that their beliefs are modified.

Another objective is to develop knowledge of mathematical concepts. This means that activities that promote PD must be guided by actions that aim for the teacher to learn the mathematics necessary for their practice from a perspective of valuing reasoning, interpretation, and production of meanings.

Developing an understanding of how students think and learn mathematics relates to the need for teachers to encourage each other to understand students' reasoning, whether through their written productions or oral communication. Students' production can be a starting point for discussion and collective reflection on their understanding of mathematics.

To develop pedagogical content knowledge, Sowder (2007), based on other authors, mentions that (prospective) teachers need to acquire knowledge of the purposes of teaching mathematics, possible students' understandings, conceptions, and misconceptions in mathematics, the curriculum, and teaching strategies and representations for teaching specific mathematics topics. Thus, this objective is related to classroom practice, where teachers must learn how specific mathematical content can be taught so students can learn.

Sowder (2007) says that to develop an understanding of the role of "equity" in mathematics, the PD should provide opportunities for teachers to recognize and address difficulties arising from social, cultural, gender, and values diversity and offer the necessary resources for equitable learning for all, regardless of social reality, sexuality, education, experiences, and adversities.

Finally, the last objective, developing a sense of identity as a mathematics teacher, concerns (prospective) teachers' need to acquire self-awareness as essential agents in the teaching and learning process. The author describes that

the process of self-reflection, which takes into account values, beliefs, concepts, emotions, and relationships, takes time and is built from different experiences with teaching and learning, being directly influenced by the feedback that the teacher receives from their students and colleagues about their work, or even from external sources, which demonstrate or do not demonstrate the recognition of teaching as a profession.

In the search to understand the PD, agreeing with Sowder (2007), Ponte and Chapman (2008) state that mathematics knowledge and knowledge of how to teach mathematics are equally important and emphasize that these two types of knowledge must be together since their initial teacher education. The authors consider this knowledge to be relevant for teachers to develop professionally. Furthermore, they understand PD according to characteristics that depend on values, habits, norms, dispositions, and, in general, the way of being a teacher.

In a context of evolution and continuity, Sancar, Atal, and Deryakulu (2021) argue that PD begins with initial teacher education, continues throughout professional life, and is influenced by the characteristics of the teacher, the content they teach and teaching strategies/methods/approaches. These authors describe variables that influence the PD of the (prospective) teacher, i.e., the way they are educated, their characteristics, what and how they teach it, how their students' performance is configured, how collaboration with their colleagues occurs, how they develop support activities, the school context, the curriculum, and reforms and policies. These variables directly influence the way the teacher learns. Therefore, they influence their PD.

In this way, different perceptions about the characterization of teachers' PD and its aspects are evident, which justifies the complexity that permeates this concept. Thus, without disregarding this diversity, we recognize that PD involves the development of knowledge, experiences, and reflections between theory and practice, starting with initial education. In this sense, we consider the teacher an active agent in the learning process.

In this context, these works describe an understanding of teachers' PD. Other authors, such as Rodrigues (2015), Estevam and Cyrino (2016), Veronez, Rodrigues, and Baldini (2023), and Veronez, Rodrigues, Galdioli, and Kowalek (2023), used some characterization of the PD to dialogue with their research. More specifically, these authors relate their research to Sowder's (2007) ideas and objectives for the PD.

Rodrigues (2015) believes teacher education is a complex process that seeks to constitute/modify elements related to the subject's beliefs, knowledge, attitudes, and conceptions. These elements are influenced by different contexts in which pre-service teachers are inserted, as well as their life history, public policies, working conditions, and cultures, among other aspects. Furthermore, the author describes that some authors, such as Sowder (2007), choose to adopt the term “professional development” to designate a specific formative process that differs from a traditional perspective.

Estevam and Cyrino (2016) are dedicated to studying the importance of encouraging and favoring teacher reflection and autonomy to recognize them as protagonists of their learning. The authors also describe that the curriculum of teaching degree courses must be (re)thought with a view to the teacher's PD.

Furthermore, Estevam and Cyrino (2016) describe the objectives for the PD of teachers who teach mathematics in the context of statistics education based on Sowder's (2007) objectives. The goals related to statistics education are to develop a shared vision for teaching and learning in the field of statistics education, a consistent understanding of statistics education for the level at which it is taught, an understanding of how students learn statistics, deep pedagogical knowledge in the field of statistics education; an understanding of the role of equity in statistics education; and a sense of self as a mathematics teacher, recognizing statistics education as a dimension of its domain.

Veronez, Rodrigues, and Baldini (2023) focused on discussing formative actions in the context of the Teaching Practicum (TPrac) during the pandemic, highlighting the educational potentials associated with the objectives for the PD of prospective mathematics teachers from Sowder's (2007) perspective. The authors consider the importance of practicum supervisors assuming their role and contributing to the education of pre-service teachers not only by indicating actions for carrying out the TPrac. They also state that these actions represent opportunities for prospective teachers to develop professionally from a learning perspective.

Veronez, Rodrigues, Galdioli, and Kowalek (2023) discuss a teacher's PD based on her narrative about developing an MM activity in a mathematics teaching degree course. The authors describe that from the teacher's experience, some elements related to MM emerged, which made it possible to associate the objectives for Sowder's PD (2007), i.e., developing a shared vision of teaching and learning, developing knowledge of mathematical content, an understanding of how students think and learn mathematics, pedagogical knowledge of content, an understanding of the role of equity in school mathematics, and a sense of identity as a mathematics teacher.

These works used the characterizations and objectives for PD according to Sowder (2007). This is where our research is situated, especially regarding configuring MM as a fruitful context for the prospective mathematics teachers' PD. Having discussed the aspects of the literature, we seek to present the research context and methodological aspects in the next section.

RESEARCH CONTEXT

This research is characterized as qualitative and comprises an empirical study whose objective is to identify elements associated with PD in the context of an MM subject in prospective teachers' education. The research scenario was a subject called MM from the perspective of mathematics education, offered to the fourth-year class of a teaching degree in mathematics at a public university in northern Paraná, where 19 students were enrolled. The subject, which has an annual offer of 60 hours of workload, distributed in 2 hours/weekly classes of 50 minutes each, was taught by the second author of the article and supervisor of this study, with the collaboration of the researcher and first author.

The syllabus for this subject contains the following description: "Mathematical modeling from the perspective of mathematics education. The phases of MM. The different perspectives and definitions of MM in mathematics

education. Development, implementation, and evaluation of MM activities aimed at the classroom. Ways of organizing and conducting an MM activity. The role of the teacher and the student in the development of an MM activity" (Universidade Estadual do Paraná, 2023, p. 32).

Based on this syllabus, the subject was organized so that students had the opportunity to experience and reflect on MM and on knowing how to do MM, inspired by the triad proposed by Dias (2005): "learning" about MM, "learning" through MM, and "teaching" using MM. About "learning" about MM, students explored different perspectives and definitions of MM in mathematics education. They came into contact with the phases of MM and the ways of organizing and conducting an MM activity. "Learning" through MM allowed students to experience MM in practice, carrying out MM activities in their formative environment. They developed, carried out, and evaluated these activities, experiencing the role of teacher and student. "Teaching" using MM took place in the subject debates that focused on using knowledge of MM to teach mathematics using MM, in other words, using it as a pedagogical alternative in class, also when invited to reflect on the role of the teacher and the student in the development of MM activities.

The organization of the subject provided pre-service teachers with an opportunity to understand the theoretical concepts that characterize MM and how it can be developed and carried out, considering its different biases. To this end, text studies and analyses of MM activities were developed and implemented. Below, we will present the methodological aspects of this research.

METHODOLOGICAL ASPECTS

Given the qualitative nature of this research, for the data production process, which took place over the two academic semesters of 2023, we resorted to collecting any information produced within the scope of the subject that constitutes our locus. Thus, our analysis material comprises the transcripts of MM classes, the answers to questions addressed to students during some of these lessons, and the *vaivém*. Each material is fundamental in the analysis process because it considers different aspects of the prospective teacher's participation in the subject and its relationship with their PD.

In the transcripts, we sought to identify elements pointed out by the students that had been made possible by the MM, which are somehow related to their education process regarding the PD. In the analysis, when referring to the transcripts, we use the letter T followed by the date of the class in which the speech occurred.

In the answers to the questions addressed to the students, we sought support to understand specific aspects that interested us. Therefore, the questions were proposed when we considered them appropriate. These questions and dates proposed are shown in Table 2. This table also indicates a code for each question.

Table 2

Questions made available to students throughout the course

Proposed questions	*Date*	*Code*
Are the mathematical approach and the model obtained efficient in responding to the proposed problem? Argue.	04/17	Q01
What do you think you learned (from and outside of mathematics and as a prospective mathematics teacher) by analyzing this mathematical modeling activity?	04/24	Q02
What do you think you learned about mathematics from this activity?	05/08	Q03
Highlight some aspects of this activity that contributed to your education process (as a teacher).	05/08	Q04
From what you studied about mathematical modeling, what do you think you learned as a prospective mathematics teacher? And what do you consider fundamental when teaching content in class?	07/03	Q05

Source: The authors (2023).

Another instrument we used in data production was the *vaivém* (back-and-forth individual feedback), which consists of a communication space (in writing) between teachers and students (individually). In the *vaivém*, the teacher asks the question(s) to the whole class, and each student answers on paper. The sheet is stored in a plastic folder with blank sheets on the front and back covers to guarantee confidentiality. Based on each student's response, the teacher asks further questions and makes comments (Silva, 2018), establishing an individualized dialogue.

In our case, communication with the *vaivém*, which started on August 21, 2023, and ended on December 11, 2023, was based on four questions: 1) What do you think is important in the development of an MM activity in class? 2) Can learning about MM help you as a teacher? How? 3) How important is MM for building students' mathematical knowledge? 4) How prepared should a teacher be to implement MM in the classroom? From there, individual dialogue started and new questions were asked, considering the students' answers to the initial questions. It is worth noting that although allowed, students did not use the *vaivém* to ask questions. In the analysis, whenever we refer to the *vaivém*, the letter V will be used, followed by the date the students answered the questions asked.

Considering data collection, we selected three students' productions for our analysis. This selection is based on the students who were most engaged in the *vaivém*. To mention these students and ensure anonymity in the analysis, we used the following fictitious names: Bellatrix, Galáxia, and Orion.

Bellatrix is a dedicated and caring student whose interest and commitment are evidenced in her active participation in mathematical modeling classes. She was not yet working as a teacher, but she participated in the debates held in class, always thinking about her future teaching activities. She had no experience with mathematical modeling but demonstrated curiosity and a willingness to learn. Always very reflective, she participated in classes, made her contributions, and asked questions.

Galáxia, although still in the process of teacher education, has two years of classroom experience. She taught 6th-grade classes and is currently teaching 9th-grade and high school classes. She has no experience with mathematical modeling. Her participation in classes throughout the course was always very active. Attentively and reflectively, she demonstrated her interest in learning.

Orion was a participatory student in mathematical modeling classes. He had not worked as a teacher yet, but whenever he had doubts in class, he would express them and actively participate in the debates that he sometimes initiated. He was enthusiastic about developing mathematical modeling activities, although inexperienced.

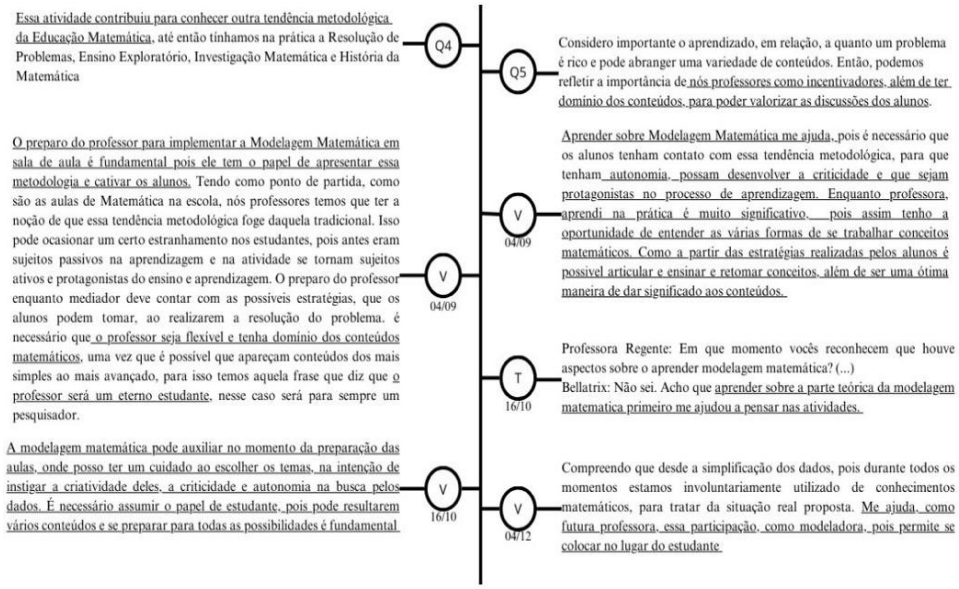
In the analysis process, looking at the data produced by these three students, we selected all the excerpts that reveal elements related to their education process, which were favored in the context of MM. Our discussions move towards relating these elements to the PD based on the objectives proposed by Sowder (2007). When presenting the students' excerpts, we indicate the analysis material in which we found them and respect the temporal issue.

PROFESSIONAL DEVELOPMENT OF PROSPECTIVE TEACHERS IN THE CONTEXT OF MATHEMATICAL MODELING

Based on the organization of the students' productions, in this section, we present the excerpts that we considered in the analysis. The presentation of such excerpts takes place in figures (Figure 1, Figure 2, and Figure 3) since we chose to observe each student independently. In presenting the excerpts, we consider the temporal issue. In other words, the excerpts appear in the chronological order of the events that generated them. To highlight some descriptions present in the excerpts, we used italics. Figure 1 contains Bellatrix's excerpts, which we analyzed.

Figure 1

Bellatrix's excerpts scheme



Source: The authors (2023).

When recording the learning about MM in her education process, Bellatrix points out that modeling helps her prepare her classes and think about how activities of this nature can be used in class. The prospective teacher believes that the process of being a teacher is not finished with the degree completion; instead, she indicates that teaching requires the teacher to continue studying, as in the excerpt: “A teacher will be an eternal student.”

The advice that teachers should continue participating in formative programs and studying to improve their knowledge of MM and other areas of mathematics education (Chaves, 2012) seems close to what Bellatrix highlights as necessary. This characteristic, pointed out by Bellatrix, is associated with the objective outlined for the PD: “To develop knowledge about mathematical concepts” because teachers can be continually engaged in learning mathematics throughout life, which means that they look for opportunities to develop their knowledge and understanding of mathematics, especially in the context of MM.

Given the scenario of her education process, Bellatrix describes some characteristics that she considers teachers must have in the context of MM, such as being flexible and encouraging. Thus, we believe that the prospective teacher recognizes that the teacher assumes this different role (Dias & Almeida, 2004) and the teacher's role is associated with the transition of leaving the center of the learning process and allowing students to become active, interested, and motivated (Cambi & Caldeira, 2023). These characteristics highlighted by Bellatrix indicate that she recognizes that the MM environment requires such characteristics from the teacher.

This recognition of how to be a teacher highlights the mobilization of specific characteristics of MM, which are related to the PD objective: “To develop pedagogical knowledge of the content” because MM provides the teacher with these characteristics, which are essential in their own nature.

The use of MM in class, considered by Bellatrix, provides an environment in which students are autonomous, develop critical thinking, and become the protagonists of their learning. Furthermore, she believes that when teachers put

themselves in students' shoes, there is an opportunity to discuss directions for using MM in class (Pollak & Garfunkel, 2013) and encourage prospective teachers to reflect on the tensions (Oliveira, 2010) they may experience. Furthermore, in this environment, teachers can manage to prepare for questions that may emerge (Meyer, Caldeira, & Malheiros, 2011) and encourage students.

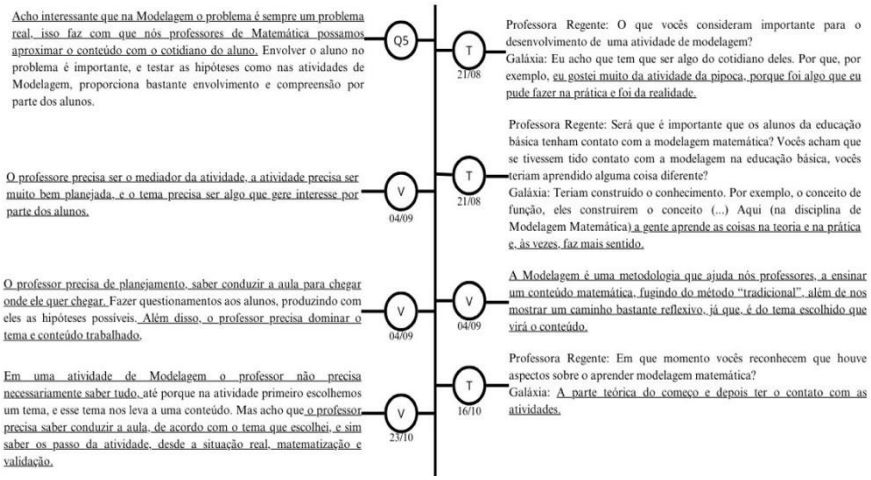
Such reflections on the teacher contribute to recognizing the different approaches to learning and students' different thinking styles. They indicate the PD objective, “To develop an understanding of how students think and learn mathematics,” because the teacher, in the context of the MM, envisions referrals for students, thinking about strategies they can develop and what interventions will be carried out in response to them. They also indicate the PD objective to “Develop an understanding of the role of “equity” in school mathematics, since the teacher considers the subjects' characteristics as influencing the way they learn.

In short, we can observe some characteristics made possible in the context of MM, as highlighted by Bellatrix, regarding her education process. The teacher must be an eternal student, have flexibility, be encouraging, develop mathematical knowledge, be prepared for students' questions, and put themselves in the students' shoes. Such characteristics, according to their attributes, relate to the objectives for the pre-service teacher Bellatrix's PD.

Next, in Figure 2, we present the Galáxia's excerpts.

Figure 2

Diagram of Galáxia's excerpts



Source: The authors (2023).

The prospective teacher highlights that learning about MM in her education process makes her classes different from traditional classes, providing an environment for students' reflection. In this sense, we understand that for her, learning about modeling encourages the teacher to reflect on their practice, valuing students' prior knowledge and their ideas (Rosa & Kato, 2014).

Galáxia believes associating theory with practice and the mathematical content taught in class with reality helps students make sense of the content. These associations favored in the MM environment can enable learning

conditions for the teacher and the student, motivating them to use real situations to explore concepts considered abstract (Rosa & Kato, 2014).

By understanding some characteristics teachers must have in the context of MM, Galáxia describes them as mediators, with knowledge of the topic and mathematical content. Thus, we understand that Galáxia thinks teachers must act as mediators since they promote collaborative work and involve students in a way that they seek their learning, creating, structuring, dynamizing, and stimulating learning situations (Rosa, 2018). Furthermore, teachers must know the proposed topic and the mathematical content to help students and provide guidance in classes in the context of modeling.

Given these characteristics of the teacher, we can relate them to two PD objectives: “Developing pedagogical knowledge of the content,” since the characteristic of being a mediator enables the development of knowledge about modeling, allowing prospective teachers to have/recognize mediation as a guiding aspect of their practice in this context; and developing mathematical knowledge associated with the objective for the PD, “Developing knowledge about mathematical concepts,” because teachers must learn the mathematics of their practice. Furthermore, mathematical learning in this context is related to the valorization of the interpretation and production of meanings inherent to the very nature of the pedagogical alternative.

The prospective teacher understands that the teacher must plan classes, using MM to direct his/her actions and those of the students toward their real objectives. These characteristics are valued in the context of MM, where the teacher's role differs (Dias and Almeida, 2004), enabling the student to become the center of learning. In this sense, we can relate Galáxia's thinking about the importance of the teacher's planning to achieve the PD objective, “To develop a shared vision of teaching and learning mathematics” because the foundation for this planning involves the teacher's actions associated with MM, evidenced in the literature and discussions within the scope of initial education. In some way, this planning, in these characteristics, relates (shares) with the discussions made by researchers in mathematics education and from curriculum perspectives.

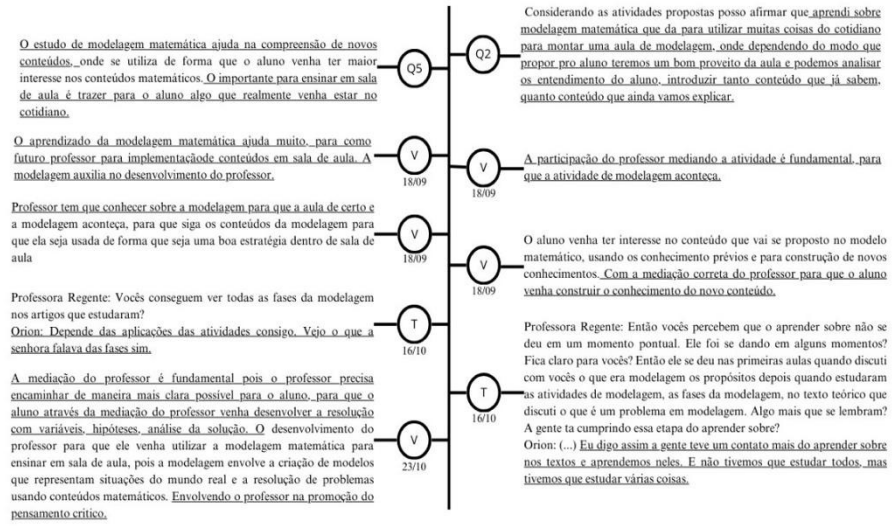
Galáxia believes teachers must know the elements of MM, such as the real situation, mathematization, and validation, as they are crucial aspects of this pedagogical alternative. Thus, we understand that these elements are essential to guide how teachers learn to act in class and choose methodological approaches for their practices (Bisognin & Bisognin, 2012).

All these characteristics highlighted by Galáxia regarding MM education, i.e., the teacher having a plan, being a mediator, developing mathematical knowledge, and developing knowledge about MM, are aligned with some of the objectives that favor the prospective teacher's PD.

Figure 3, shown below, contains Orion's excerpts.

Figure 3

Orion's excerpts scheme



Source: The authors (2023).

The prospective teacher believes that learning about MM in his education process helps prepare his classes, as this methodology provides a relationship between mathematics and the student's reality. We consider that this relationship, using real situations to explore concepts considered abstract (Rosa & Kato, 2014), provided within the scope of MM, is essential for him since it highlights, at various times, its relevances.

We associate Orion's idea about the importance of preparing his class with the PD objective “To develop a shared vision of teaching and learning mathematics” since the research developed by the teacher, their studies, and relationships with other peers influence how they prepare for the class, as well as how they develop it. This necessarily involves negotiating the meanings of their realities with their peers' realities, whether they are from literature, their workmates, their trajectory, etc.

When reporting that teachers should have specific attitudes in the context of MM to result in “a good use of the class,” Orion describes that they must develop some characteristics, such as mediating mathematical content. In this sense, we understand that, for Orion, teachers must be mediators because they must assist in students' collaboration and engagement by encouraging them to seek learning through creating, structuring, dynamizing, and stimulating learning situations (Rosa, 2018). Furthermore, teachers must master mathematical content to support students and guide classes within the context of MM.

Given the characteristics Orion underscored, we relate teachers as mediators to the PD objective, “Developing pedagogical knowledge of the content,” since mediation is an essential characteristic for the use of MM as a pedagogical alternative that is necessary for its progress and developing mathematical knowledge to the PD objective “Developing knowledge about mathematical concepts,” since the teacher needs to know mathematics to use it in their practices.

Orion considers that teachers, as mediators, promote students' construction of new knowledge and mobilization of previously learned concepts during the development of MM activities. Furthermore, he highlights that MM provides

teachers with a learning process, as they need to know about the methodology to develop their critical thinking and achieve their objectives in the classroom environment. Furthermore, learning about MM is associated with the PD objective “To develop pedagogical knowledge of the content” since it is necessary to learn about it to use it.

Thus, we consider that these were the characteristics made possible in the context of MM, as evidenced by Orion’s speech about his education process: preparing his class, being a mediator, developing mathematical knowledge, and developing knowledge about MM.

In Table 3, we systematize aspects related to the prospective teachers' education highlighted in the context of MM, associated with the PD objectives from Sowder's perspective (2007).

Table 3

Summary of results regarding the prospective teachers' education and objectives for professional development

Prospective teacher	Aspects related to the prospective teachers' education highlighted in the context of mathematical modeling	Objectives for professional development (Sowder, 2007)
Bellatrix	Teacher be an eternal student develop mathematical knowledge	"Develop knowledge of mathematical concepts"
	Be flexible, be encouraging	"Develop pedagogical content knowledge"
	Be prepared for students' inquiries	"Develop an understanding of how students think and learn mathematics"
	Put oneself in the students' shoes	"Develop an understanding of the role of "equity" in school mathematics"
Galáxia	Teacher must have a plan	"Develop a shared vision of teaching and learning mathematics"
	Teacher must be mediator	"Develop pedagogical content knowledge"
	Develop mathematical knowledge	"Develop knowledge of mathematical concepts"
Orion	Preparing class	"Develop a shared vision of teaching and learning mathematics"
	Develop knowledge about mathematical modeling	
	Be a mediator	"Develop pedagogical content knowledge"
	Develop mathematical knowledge	"Develop knowledge of mathematical concepts"

Source: The authors (2023).

Bellatrix, Galáxia, and Orion emphasize some characteristics that relate to the importance of the teacher being an eternal student, developing both mathematical knowledge and having an understanding of the role of equity in mathematics education, being a mediator, and planning classes. All these characteristics, in general, reveal that the formative context in MM helped prospective teachers to think about aspects related to their education, and, in this sense, they had the opportunity to develop professionally. Below, we present our final considerations.

FINAL CONSIDERATIONS

In this study, we seek to identify elements associated with PD in the context of an MM subject in prospective teachers' education. To this end, we considered three students (Bellatrix, Galáxia, and Orion) in our analysis.

In particular, Bellatrix emphasizes how important it is that teachers are lifelong learners, developing both deep mathematical knowledge and an understanding of the role of equity in mathematics education. Galáxia highlights the need for lesson planning and effective mediation in teaching, focusing on developing pedagogical knowledge of mathematical content and concepts. In turn, Orion emphasizes careful lesson preparation, the development of a shared vision of teaching and learning mathematics, and the ability to mediate students' learning process, integrating MM meaningfully. All these manifestations demonstrate fundamental characteristics for the PD of the prospective teachers, which are associated with the objectives proposed by Sowder (2007).

By demonstrating that they understand the importance of MM in their education for teaching, these three prospective teachers recognize that learning about this methodology prepares them for future classes and develops skills such as creating reflective and participatory learning environments.

The fact that they believe that the teacher must be an eternal student, be flexible, be a mediator, and have a solid command of mathematical content reveals that they understand that the teacher's role goes beyond simply transmitting knowledge, favoring students' construction of meanings and creating spaces for them to become protagonists of their own learning. The indication of these elements highlights a series of characteristics associated with MM and, at the same time, are related to the PD objectives indicated by Sowder (2007). As one of the aspects of their PD, they point out that teaching practice is a process in constant evolution and that it is essential to continue studying and improving throughout one's career.

These prospective teachers also value the integration of specific and pedagogical knowledge. They understand that it is essential to establish connections between theory and practice and between mathematical content and students' reality. In short, Bellatrix, Galáxia, and Orion are committed to improving their teaching practice. They recognize that MM offers unique opportunities to engage students, promote autonomy, and develop critical thinking skills.

Our research describes a detailed and contextualized analysis of MM as a tool for teacher PD. By integrating theory and practice, exploring the specific application of MM, and investigating its impact on equity and professional identity, we offer new perspectives and contributions to the field. These advances complement the existing literature and offer practical and theoretical directions for future research and practice in teacher education.

For future research, we suggest researchers explore teacher education in other subjects that consider MM practices. One possibility is to specifically explore aspects related to the teacher's PD regarding self-knowledge promoted in the context of MM.

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