UMA NOTA SOBRE ANALITICIDADE PARA SISTEMAS PIEZOELÉTRICOS

Hugo D. Fernández Sare[1], **Bernardette Miara**[2], **Mauro L. Santos**[3]

1.- Department of Research and Development, National Laboratory for Scientific Computation. Rua Getulio Vargas 333, Quitandinha CEP 25651-070. Petrópolis, Rio de Janeiro-Brazil. e-mail: hugo@lncc.br

2.- Département de Modélisation et Simulation Numérique École Supérieure d'Ingénieurs en Électrotechnique et Électronique, 2 Boulevard Blaise-Pascal, 93160 Noisy-le-Grand, France e-mail: miarab@esiee.fr

3.- Faculdade de Matemática, Universidade Federal do Pará, Campus Universitario do Guamá, Rua Augusto Corrêa 01, Cep 66075-110, Pará, Brazil e-mail: ls@ufpa.br

Resumo- Neste trabalho consideramos um sistema piezoelétrico dissipativo no qual o termo dissipativo é dado através da parte mecánica do tensor de estresse. O problema de Cauchy abstrato associado a este sistema gera um semigrupo de operadores lineares. Mostraremos a analiticidade deste semigrupo o qual, a sua vez, implica o decaimento exponencial da energia correspondente e a estabilidade do sistema.

Palavras-chave: Materiais piezoelétricos, dissipação de estresse, semigrupo analítico.

A NOTE ON ANALYTICITY TO PIEZOELECTRIC SYSTEMS

Abstract- In this paper we consider a piezoelectric dissipative system in which a dissipation term is given through the mechanical part of the stress tensor. The abstract Cauchy problem associated to this system generates a semigroup of linear operators. We establish the analyticity of this semigroup which, in turns, implies the exponential decay of the corresponding energy and the stability of the system.

KeyWord: Piezoelectric materials, stress dissipation, analytic semigroup.

1. INTRODUÇÃO

Theoretical aspects of control and stabilization of piezoelectric bodies have received great attention in the last years. Let us mention the most recent results. Exact controllability, when a piezoelectric control is applied on the whole boundary, has been studied in the case of three-dimensional structures [10, 12] and for shells [11] . Stabilization due to boundary dissipation for layered structures have also been addressed [4, 5], other references can be found in these papers. More recently the exponential stability of a piezoelectric system has been established by B. Miara and M. L. Santos [13] who considered mechanical dissipation induced by the elastic displacement. In this work a new mechanical dissipation effect is taken into account, it is due to the stress tensor. Our main result is to prove the analyticity of the semigroup of linear operators associated to piezoelectric systems, this, in particular, implies the exponential stability of the associated energy and also the so-called spectrum determined growth property (SDG-property) of the corresponding semigroup. For this result the main tool is the characterization given by Lemma **1** proved in [8]. More details about analyticity are presented in [2, 3, 6, 7, 8, 14] and in the references therein.

2. MODELO FÍSICO

When subjected to applied mechanical force f and electric charges g this body undergoes a piezoelectric displacement formed by an elastic displacement $\mathbf{u}(x,t) = (u_i(x,t)) : \overline{Q} \longrightarrow R^3$ and an electric potential $\varphi(x,t) : \overline{Q} \longrightarrow R$ given formally by the following evolution equations:

$$\left\{ \begin{array}{rcll} \tau \mathbf{u}_{tt} - \mathrm{div}\mathbf{T}(\mathbf{u},\varphi) - \gamma \mathrm{div}\mathbf{C}(\mathbf{u}_t) & = & \mathbf{f} & \text{in} \quad Q, \\ -\mathrm{div}\mathbf{D}(\mathbf{u},\varphi) & = & g & \text{in} \quad Q, \end{array} \right. \tag{1}$$

associated to Cauchy initial conditions

$$\mathbf{u}(x,0) = \mathbf{u}^0(x), \quad \mathbf{u}_t(x,0) = \mathbf{u}^1(x) \quad \text{in} \quad \Omega, \tag{2}$$

and homogeneous Dirichlet boundary conditions

$$\mathbf{u} = \mathbf{0} \quad \text{and } \varphi = 0 \quad \text{on} \quad \Sigma. \tag{3}$$

We recall the notation

$$\partial_k \cdot = \frac{\partial \cdot}{\partial x_k}, \quad (\text{div}\mathbf{T})^i = \partial_k T^{ki}, \quad \text{div}\mathbf{D} = \partial_k D^k,$$

and

$$\mathbf{u}_t = \frac{\partial \mathbf{u}}{\partial t}.$$

First we will re-write our problem as an evolution system. To this aim, note that the elastic displacement field $\mathbf{u}$ is solution to the evolution problem

$$\begin{cases} u_{j,tt} - \frac{\partial}{\partial x_i}\left\{c^{ijkl}s_{kl}(\mathbf{u}) + e^{kij}\frac{\partial}{\partial x_k}(\widetilde{\varphi}(\mathbf{u}))\right\} - \cdots \\ \qquad \cdots - \gamma\frac{\partial}{\partial x_i}\left\{c^{ijkl}s_{kl}(\mathbf{u_t})\right\} = 0, \\ \mathbf{u}(x,0) = \mathbf{u}^0(x), \quad \mathbf{u}_t(x,0) = \mathbf{u}^1(x), \\ \mathbf{u} = \mathbf{0}. \end{cases}$$

Let us introduce the complex valued Hilbert space

$$\mathcal{H} = \mathbf{H}_0^1(\Omega) \times \mathbf{L}^2(\Omega)$$

and the unbounded linear operator $\mathcal{A}$ given by

$$\mathcal{A} : \mathbf{U} = (\mathbf{u}, \mathbf{v})' \in \mathcal{H} \longrightarrow \left(v_j, \frac{\partial}{\partial x_i}\left\{c^{ijkl}s_{kl}(\mathbf{u}) + \cdots \right.\right.$$

$$\left.\left. e^{kij}\frac{\partial}{\partial x_k}(\widetilde{\varphi}(\mathbf{u}))\right\} - \gamma\frac{\partial}{\partial x_i}\left\{c^{ijkl}s_{kl}(\mathbf{v})\right\}\right)'$$

2. RESULTADOS

Now we will establish and we will prove the analyticity of the semigroup formulated in the previous subsection. Our main tool will be the well known result about analytic semigroups, see for example [1, 8, 9, 15].

Lemma 1.- *Let $\rho(\mathcal{A})$ be the resolvent set of the linear operator $\mathcal{A}$. Then, a semigroup of contractions $\{e^{t\mathcal{A}}\}_{t\geq 0}$ in a Hilbert space $\mathcal{H}$ with norm $\|\cdot\|_{\mathcal{H}}$ is of analytic type if and only if*

$$iR \subset \rho(\mathcal{A}) \qquad (i := \sqrt{-1}\,) \tag{4}$$

and

$$\limsup_{|\lambda|\to\infty} ||\lambda(i\lambda I - \mathcal{A})^{-1}||_{\mathcal{L}(\mathcal{H})} < \infty. \tag{5}$$

Using this Lemma, our main theorem is the following

Theorem 2.- *The semigroup generated by the linear operator $\mathcal{A}$ associated to system (1) is of analytic type.*

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